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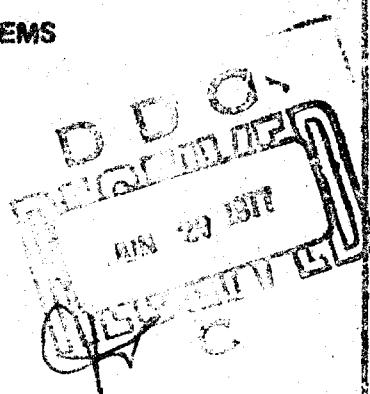


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A FORTRAN IV COMPUTER PROGRAM FOR THE TIME DOMAIN ANALYSIS
OF THE TWO-DIMENSIONAL DYNAMIC MOTIONS
OF GENERAL BUOY-CABLE-BODY SYSTEMS

by

Henry T. Wang



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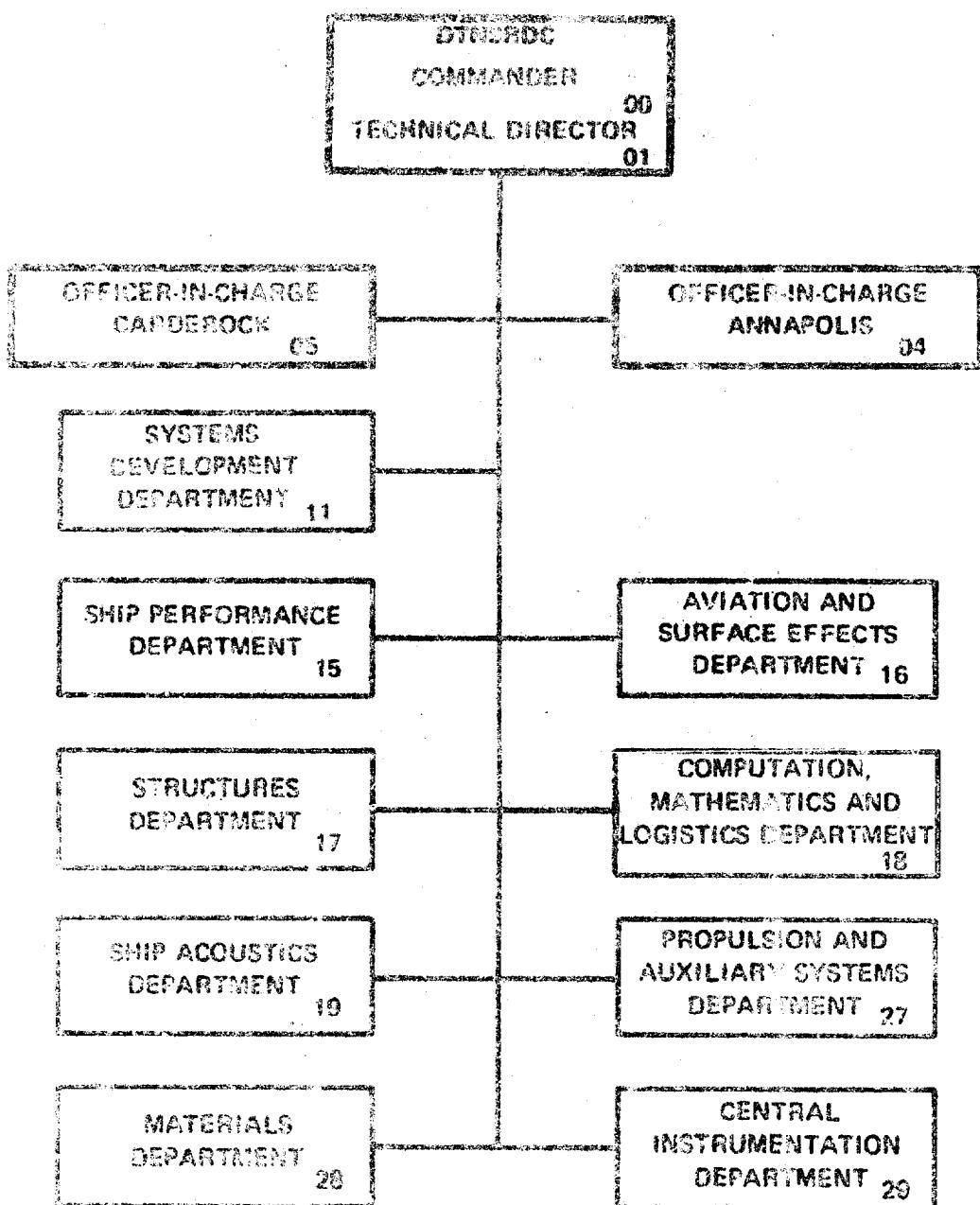
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1000

June 1977

Report 77-0043

MAJOR TNSRDC ORGANIZATIONAL COMPONENTS



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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER DTNSRDC-77-004n	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (Subtitle) A FORTRAN IV COMPUTER PROGRAM FOR THE TIME DOMAIN ANALYSIS OF THE TWO-DIMENSIONAL DYNAMIC MOTIONS OF GENERAL BUOY-CABLE-BODY SYSTEMS		5. TYPE OF REPORT & PERIOD COVERED
7. AUTHOR(s) Henry T. Wang		6. PERFORMING ORG REPORT NUMBER
8. CONTRACT OR GRANT NUMBER(s)		
9. PERFORMING ORGANIZATION NAME AND ADDRESS David W. Taylor Naval Ship Research and Development Center Bethesda, Maryland 20084		10. PROGRAM ELEMENT PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element No 62711N Task Area WF 11 121 703 Work Units 1-1552-130, 1-1552-145
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Air Development Center Code 20013 Johnsville, Warminster, Pa. 18974		12. REPORT DATE June 1977
14. MONITORING AGENCY NAME & ADDRESS IF different from Controlling Office		13. NUMBER OF PAGES 96
16. DISTRIBUTION STATEMENT (of this Report)		15. SECURITY CLASS (of this report) UNCLASSIFIED
		16a. DECLASSIFICATION/DOWNGRADING SCHEDULE
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Two-Dimensional Dynamic Motions Time Domain Buoy-Cable-Body System Computer Program		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The present report gives a detailed description of Program CABUOY which analyzes in the time domain the two-dimensional dynamic behavior of general ocean cable systems consisting of a surface buoy, connecting cable, and intermediate bodies. The equations which model the motions of the surface waves and the various components of the cable system are presented, and the subroutines of the program are briefly outlined. Instructions on use of the program include a listing of the input READ statements, definitions of the input variables, and a number of comments on the entering of input data. Several sample problems are given to illustrate use of the program, the output of the program, and computer costs for a range of cases. The listing of the program is given in the appendix		

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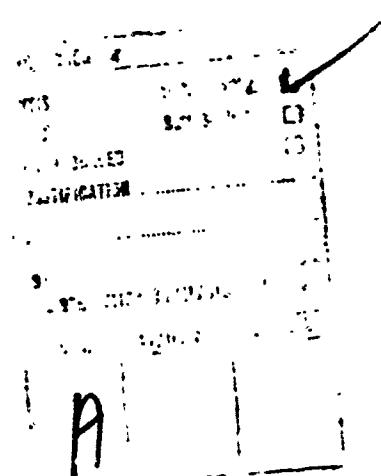
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ABSTRACT

The present report gives a detailed description of Program CABOY, which analyzes in the time domain the two dimensional dynamic behavior of general ocean cable systems consisting of a surface buoy, connecting cable, and intermediate bodies. The equations which model the motions of the surface waves and the various components of the cable system are presented, and the subroutines of the program are briefly outlined. Instructions on use of the program include a listing of the input READ statements, definitions of the input variables, and a number of comments on the entering of input data. Several sample problems are given to illustrate use of the program, the output of the program, and computer costs for a range of cases. The listing of the program is given in the appendix.

ADMINISTRATIVE INFORMATION

The work described in this report was authorized by the Naval Air Development Center under Project Orders 4-0601 and 0-0611 respectively dated 5 March 1974 and 13 February 1976. The work was performed under internal Work Units I-1552-130 and I-1552-145.

INTRODUCTION

The dynamic motion characteristics of cable systems are currently of extreme interest, and several major surveys of cable dynamics studies have been made in recent years¹⁻³.

An ocean cable system generally consists of the following three components:

1. A ship or surface float at the upper end.
2. A cable whose properties may vary along its length.
3. Intermediate bodies along the cable, including the possibility of a body at the lower end.

Previous studies have usually focused on only one of the above components. For example, the principal emphasis in many studies is on the dynamic characteristics of the cable itself.

¹Dillon, D.B., "An Inventory of Current Mathematical Models of Scientific Data - Gathering Moors," Hydrospace-Challenger, Inc. TR 4450 0001 (Feb 1973).
A complete list of references is given on pages 61-63.

²Choo, Y.I. and M.J. Casarella, "A Survey of Analytical Methods for Dynamic Simulation of Cable-Body Systems," Journal of Hydraulics, Vol. 7, No. 4, pp. 137-144 (Oct 1973).

³Albertsen, N.D., "A Survey of Techniques for the Analysis and Design of Submerged Moving Systems," Civil Engineering Laboratory Technical Report R-815 (Aug 1974).

Conditions at the ends of the cable are then either those of prescribed motions or simple representations of the surface buoy or lower body. It appears that such studies were carried out mainly to demonstrate the feasibility of a particular method of solving for the dynamic characteristics of the cable. Choo and Casarella² discuss the merits and drawbacks of the three principal analytical methods: linearized frequency-domain method, method of characteristics, and finite element method. In other studies, the principal emphasis is on the dynamic characteristics of the surface buoy or the lower body and the effect of the cable is then approximated in various ways. It is clear that these studies are suitable only for analyzing particular types of cable systems, also, only the dynamic characteristics of certain components are accurately described.

The present report gives details on Program CABUOY, which analyzes in the time domain the two-dimensional dynamic behavior of all three components of a general cable system. This program has already been briefly described.⁴ Although it was developed principally to analyze the dynamic behavior of sonobuoy systems, for which it is of interest to know the dynamic behavior of the surface buoy, connecting cable, and lower acoustic detection units, the great generality and versatility of the program make it useful for a wide variety of other cable systems.

The report first presents in detail the equations which form the basis of the program. These include equations for the steady-state cable configuration and for the dynamic motions of the surface waves, surface buoy, cable, and intermediate bodies. Each of the program subroutines is briefly described. Detailed input instructions include a listing of the input FORTRAN READ statements, definition of the input variables contained in these READ statements, and comments on the entry of input data. Several sample problems serve to illustrate program use, output, and computer costs. The program is listed in the appendix.

Both the input instructions and the sample problems illustrate the wide applicability of the program. The sample problems range from a parametric study of the accuracy and computer cost of various finite element representations of the cable to the analysis of a complete buoy-cable-body system moored in the presence of typical ocean waves and current profiles. The characteristics of each component of the ocean surface waves may be specified by the user or may be internally generated by the program by means of the Pierson-Moskowitz spectrum. The surface buoy at the top of the cable may have a relatively wide range of sizes and shapes. It may be a prolate or oblate spheroid of any aspect ratio provided its horizontal

⁴Wang, H.T., "Preliminary Report on a Fortran IV Computer Program for the Two-Dimensional Dynamic Behavior of General Ocean Cable Systems," DTNSRDC Departmental Report SPD-633-01 (Aug 1975).

length is small compared to the wavelengths of the significant ocean waves, or it may be a spar buoy of any size. The reasons for these particular choices of buoy shapes and sizes are given in the section on surface buoys. Alternatively, motions may be prescribed at the upper end. The user determines the accuracy and computer cost of the dynamic analysis of the cable by specifying the total number of cable segments as well as the length of each segment. Several different formulations are given for the added masses and drag coefficients of the intermediate bodies.

STEADY-STATE CALCULATIONS

CABLE EQUATIONS

The program first calculates the configuration of the cable system in the presence of a steady-state current* alone, in the absence of any time-dependent excitations. The differential equations for the steady-state configuration of the cable are well known and take the following form for the coordinate system shown in Figure 1; for example, see Springston.⁵

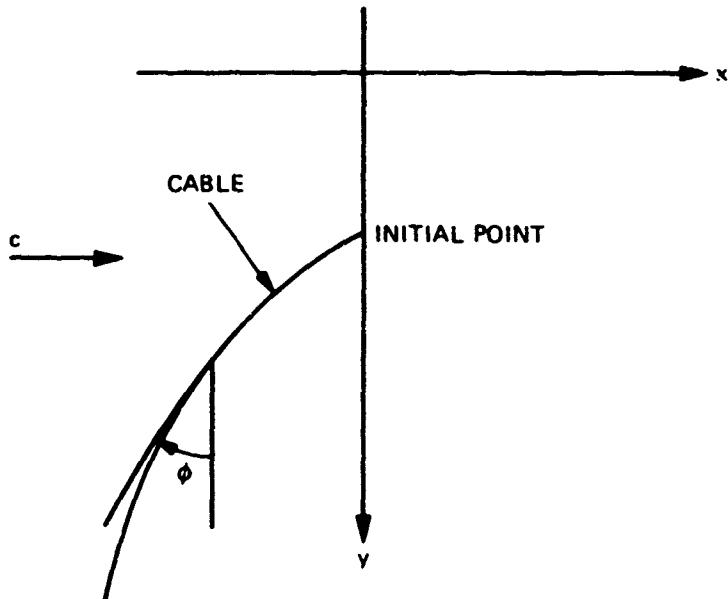


Figure 1 – Definition of Coordinate System

*The term current is used to denote the steady-state fluid velocity relative to the cable. For the case of a cable towed in still water, the current has magnitude equal to and direction opposite to the towing velocity.

⁵Springston, G.B. Jr., "Generalized Hydrodynamic Loading Functions for Bare and Faired Cables in Two-Dimensional Steady-State Cable Configurations," NSRDC Report 7424 (Jun 1967).

$$-T \frac{d\phi}{ds_0} + l + \sin \phi W = 0 \quad (1)$$

$$\frac{dT}{ds_0} + G + \cos \phi W = 0 \quad (2)$$

$$\frac{ds}{ds_0} = 1 + \epsilon \quad (3)$$

$$\frac{dx}{ds_0} = (1 + \epsilon) \sin \phi \quad (4)$$

$$\frac{dy}{ds_0} = (1 + \epsilon) \cos \phi \quad (5)$$

- where T = cable tension
 ϕ = angle of the cable segment with the vertical (see Figure 1)
 s_0 = reference cable length (when $T = T_0$) measured from the initial point
 T_0 = reference tension
 I, G = normal and tangential drag forces, respectively, per unit length acting on the cable
 W = weight in fluid per unit length of the cable
 s = stretched cable length measured from the initial point
 ϵ = cable strain; $\epsilon = 0$ at $T = T_0$
 x = horizontal displacement, positive to the right
 y = vertical displacement, positive downward

For smooth, approximately round cables, the normal and tangential drags may be taken as respectively proportional to the squares of the velocities normal and tangential to the cable⁵

$$I = \frac{1}{2} \rho C_D d c_n |c_n| \quad (6)$$

$$G = \frac{1}{2} \rho C_T d c_t |c_t| \quad (7)$$

- where ρ = fluid density
 C_D, C_T = normal and tangential drag coefficients respectively
 d = cable diameter
 c_n = component of the current normal to the cable = $c \cos \phi$
 c_t = component of the current tangential to the cable = $-c \sin \phi$
 c = magnitude of the current, taken to act only in the x -direction

The tension-strain function is assumed to be of the form⁶

$$T - T_0 = C_1 \epsilon^{C_2} \quad (8)$$

where C_1 = constant of elasticity; $C_1 = AE$ for a linearly elastic cable
 A = cross-sectional area = $\pi d^2/4$ for a round cable
 E = modulus of elasticity
 C_2 = an exponent; $C_2 = 1$ for a linearly elastic cable

Equation (8) enables a nonlinear tension-strain relation to be modeled by only two input variables, C_1 and C_2 . It is more convenient to express ϵ as a function of $(T - T_0)$ in order to eliminate it in Equations (3) through (5):

$$\epsilon = \left(\frac{T - T_0}{C_1} \right)^{1/C_2} \quad (9)$$

INTERMEDIATE BODIES

It is assumed that conditions at the top of the cable are known and the integration of Equations (1) through (5) proceeds down the cable. At an intermediate body, the integration must be interrupted and the unknown cable variables T_u and ϕ_u below the body must be related to the known variables T_k and ϕ_k above the body. Reducing the three-dimensional equations contained in References 7 and 8 to the present two-dimensional case results in the following two equations for T_u and ϕ_u :

$$T_u = \sqrt{(\sin \phi_k T_k + D_x)^2 + (-\cos \phi_k T_k + W_B)^2} \quad (10)$$

$$\phi_u = \tan^{-1} \frac{\sin \phi_k T_k + D_x}{-\cos \phi_k T_k + W_B} \quad (11)$$

where D_x is the drag on the body and

⁶Thresher, R.W. and J.H. Nath, "Anchor-Last Deployment Procedure for Mooring," Oregon State University Report 73-5 (Jun 1973).

⁷Wang, H.T., "Effect of Nonplanar Current Profiles on the Configuration of Moored Cable Systems," NSRDC Report 3692 (Oct 1971).

⁸Wang, H.T., "A FORTRAN IV Program for the Three-Dimensional Steady-State Configuration of Extensible Flexible Cable Systems," NSRDC Report 4384 (Sep 1974).

$$D_x = \frac{1}{2} \rho C_D A_x c |c| \quad (12)$$

Here also, $C_D A_x$ is the drag area of the body for flow in the x -direction and W_B is the weight of the body in fluid.

BOUNDARY CONDITIONS

Initial Value Cases

The integration of the above differential equations is most convenient when the tension T and the angle ϕ are known at one end of the cable. These are known for certain cases of single-point moored cables and towing cables. For the moored cases, the program starts with the known conditions at the top of the cable and integrates the five differential equations until the lower end of the cable is reached. This is the simplest case for the program since the numbering of the cable segments and intermediate bodies starts at the top of the cable. For towing cable cases, where the conditions are known at the lower towed body, the program integrates the differential equations twice. They are first integrated from the towed body to the upper point, thus fixing the conditions at this point. Then, in order to conform to the numbering system which is used for the dynamic calculations, the equations are integrated once again from the upper point down to the lower towed body.

Boundary Value Cases

In many applications, the values of T and ϕ are not known *a priori* at any point along the cable. It is necessary to treat these cases as boundary value problems and use iteration techniques to obtain the solution. The present program contains the iteration schemes for two cases of particular interest for oceanbuoy systems - a cable of given length moored in a given ocean depth⁹ and a free-floating cable system.¹⁰ The iteration subroutine is written so that the user may conveniently implement iteration schemes for other applications.

⁹Wang, H.T. and B.L. Webster, "Current Profiles Which Give Rise to Nonunique Solutions of Moored Cable Systems," Paper OTC 1538, Fourth Annual Offshore Technology Conference, Houston, Texas (May 1972).

¹⁰Wang, H.T. and T.L. Moran, "Analysis of the Two-Dimensional Steady-State Behavior of Extensible Free-Floating Cable Systems," NSRDC Report 3721 (Oct 1971).

OCEAN SURFACE WAVES

DESCRIPTION OF MOTION

For ocean depths greater than one-half the wavelength,¹¹ the water particle trajectories due to a single progressive wave are, according to linearized first-order theory, given by^{11,12}

$$x_w = a_w e^{-ky} \cos(kx - \sigma t + \theta_w) \quad (13a)$$

$$y_w = -a_w e^{-ky} \sin(kx - \sigma t + \theta_w) \quad (13b)$$

where x_w, y_w = water particle displacements in the (x, y) directions, respectively

a_w = wave amplitude

k = wave number = $2\pi/\lambda$

λ = wavelength

σ = circular frequency = $\sqrt{2\pi g/\lambda} = 2\pi f$

f = frequency

t = time

g = gravity constant = $32.2 \text{ ft/sec}^2 (9.81 \text{ m sec}^2)$

θ_w = phase angle

It is of interest to note that the trajectories describe circular orbits with a radius which decays exponentially with depth.

For an irregular sea consisting of N distinct components, the resultant water particle displacements are obtained by a summation of the above expressions, resulting in

$$x_w = \sum_{i=1}^N a_{wi} e^{-k_i y} \cos(k_i x - \sigma_i t + \theta_{wi}) \quad (14a)$$

$$y_w = -\sum_{i=1}^N a_{wi} e^{-k_i y} \sin(k_i x - \sigma_i t + \theta_{wi}) \quad (14b)$$

Differentiations with respect to time yield the following results for water particle velocities and accelerations

¹¹ Lamb, H., "Hydrodynamics," Sixth Edition, Dover Publications, New York (1945), pp. 363-370, pp. 152-155.

¹² Wehausen, J.V. and E.V. Laitone, "Surface Waves," in "Handbuch der Physik," Vol. 9, Springer Verlag, Berlin (1960), pp. 446-778.

$$\dot{x}_w = \sum_{i=1}^N \sigma_i a_{wi} e^{-k_i y} \sin(k_i x - \sigma_i t + \theta_{wi}) \quad (15a)$$

$$\dot{y}_w = \sum_{i=1}^N \sigma_i a_{wi} e^{-k_i y} \cos(k_i x - \sigma_i t + \theta_{wi}) \quad (15b)$$

$$\ddot{x}_w = \sum_{i=1}^N -\sigma_i^2 a_{wi} e^{-k_i y} \cos(k_i x - \sigma_i t + \theta_{wi}) \quad (16a)$$

$$\ddot{y}_w = \sum_{i=1}^N \sigma_i^2 a_{wi} e^{-k_i y} \sin(k_i x - \sigma_i t + \theta_{wi}) \quad (16b)$$

CHOICE OF COMPONENTS

The computer program allows the user two options for describing an irregular sea. He may specify the values of N , a_{wi} , σ_i , and θ_{wi} or he may use an energy spectrum $S_S(\sigma)$ to define the wave amplitudes

$$a_{wi} = \sqrt{S_S(\sigma_i) \Delta \sigma} \quad (17)$$

where $\sigma_i = \sigma_{i-1} + \Delta \sigma$

$$\Delta \sigma = (\sigma_u - \sigma_l)/N$$

σ_u = upper limit of the significant range of σ 's

σ_l = lower limit of the significant range of σ 's

The program uses the Pierson-Moskowitz energy sea spectrum of the form

$$S_S(\sigma) = \frac{A}{\sigma^5} e^{-B/\sigma^4} \quad (18)$$

where $A = 0.0081 g^2$ and $B = 33.56 h_{1/3}^2$. Here $h_{1/3}$ is the significant wave height, the average of the one-third highest peak-to-trough heights. As reported by Frank and Salvesen,¹³ the 11th International Towing Tank Conference (Tokyo) (1966) recommended the spectrum in this form for computations when information is not available on typical sea spectra.

¹³Frank, W. and N. Salvesen, "The Frank Close-Fit Ship-Motion Computer Program," NSRDC Report 3289 (Jun 1970).

Other forms for $S_g(\sigma)$ can, of course, be conveniently programmed. Values for $h_{1/3}$, σ_u , and σ_g for State 0 to 9 seas may be found in Table 1 of Reference 13. The program sets the values of θ_{wi} to be evenly spaced from θ_{w1} to $360-\theta_{w1}$ degrees.

PRESCRIBED SURFACE MOTIONS

The program allows the user either to prescribe the motion at the surface or to describe it by means of differential equations of motion for a surface buoy.

If the surface buoy or ship is sufficiently large that its motions are not appreciably affected by the presence of the cable, these motions may be calculated separately and used as input for the present program. Several programs are available to calculate the pitch and heave motion responses of surface ships, for example, the Frank Close-Fit Ship Motion Computer Program.¹³ This approach is also valid for laboratory simulations of cable dynamics where the motions at the upper end of the cable are often prescribed. A third area of application could be a full-scale trial where the motions of the surface ship or platform can be readily measured.

The program considers the prescribed motion of the end of the cable as composed of a series of sinusoidal components in the horizontal and vertical directions.

$$x_s = \sum_{i=1}^N a_{xi} \cos(-2\pi f_i t + \theta_{si}) \quad (19)$$

$$y_s = \sum_{i=1}^N -a_{yi} \sin(-2\pi f_i t + \theta_{si}) \quad (20)$$

where x_s, y_s = horizontal and vertical components of the surface motion, respectively

a_{xi}, a_{yi} = amplitudes for the i th component of x_s and y_s , respectively

f_i, θ_{si} = frequency and phase angle for the i th component, respectively

If so desired, other forms for the prescribed motion may be conveniently added to the program, e.g., other functions of time such as powers of t or exponentials.

SURFACE BUOY EQUATIONS

GENERAL CONSIDERATIONS

It is well known that the added mass and damping coefficients of surface buoys are, in general, functions of the frequency of the oscillation.^{14 15} In the time domain, this requires the solution of integrodifferential equations which contain convolution integrals. Alternatively, if the frequency-dependent coefficients can be expressed as simple polynomials of the frequency, the integrodifferential equations may be replaced by a set of higher order differential equations.¹⁴ In either case, the solutions are complex and/or time-consuming in the time domain. Thus, surface buoy motions have usually been solved in the frequency domain. In this approach, the steady-state harmonic response is obtained for each frequency component of the exciting surface waves. The total response to the sum of the individual wave components is then obtained by linear superposition. Experiments have shown that this procedure generally yields satisfactory results for pitch and heave motions of surface ships.

Because of difficulties in solving buoy equations in the time domain, the frequency domain approach has also been used to study the motion of cable-buoy systems. Perhaps the most comprehensive of these studies is the Goodinan et al. computer program¹⁶ which considers four different buoy shapes. In addition to facilitating the solution for general buoy shapes, the frequency domain approach has the additional advantages of immediately giving the steady-state harmonic response (no need to wait for the transient response to die down) and of reducing the computer time required to obtain cable motions.¹⁷ However, the drawbacks to this approach include neglect of all nonlinearities and the assumption that all the dynamic response variables are small compared to their steady-state values. This approach would not be able to predict, for example, the large dynamic snap loads which occur when the cable goes slack.

In view of the above drawbacks and also in view of the existence of the comprehensive frequency-domain computer program described in Goodman et al.,¹⁶ it was decided to use a

¹⁴Tick, L.J., "Differential Equations with Frequency-Dependent Coefficients," *Journal of Ship Research*, Vol. 3, No. 2, pp. 45-46 (Oct 1959).

¹⁵Ogilvie, T.F., "Recent Progress toward the Understanding and Prediction of Ship Motions," Fifth Symposium of Naval Hydrodynamics, Bergen, Norway, pp. 3-128 (Sep 1964).

¹⁶Goodman, T.R. et al., "Static and Dynamic Analysis of a Moored Buoy System," National Data Buoy Center Report 6113.1 (Apr 1972).

¹⁷Wang, H.T., "A Two-Degree-of-Freedom Model for the Two-Dimensional Dynamic Motions of Suspended Extensible Cable Systems," NSRDC Report 3663 (Oct 1971).

time domain approach in the present study. In order to make this approach feasible, it was important to find classes of buoys which did not have frequency-dependent added mass and damping coefficients. A literature search revealed two such classes: spar buoys^{18,19} and small buoys. Spar buoys are buoys with circular cross sections and large draft-to-diameter ratios H/d. Because of their slenderness, the added inertia terms of these buoys are essentially those for infinite fluid, and the frequency-dependent wave damping coefficients are zero to first order approximation.¹⁸ The other class corresponds to buoys whose typical dimension a is so small compared to the ocean wavelengths λ that the reduced frequency $\bar{\sigma}$ given by

$$\bar{\sigma} = 2\pi \frac{d}{\lambda} \ll 1 \quad (21)$$

is much less than unity for the range of λ values corresponding to ocean waves of interest.

For surface buoys of sonobuoy systems, whose typical dimension is of the order of 1 ft (0.305 m), the above condition holds for the large majority of sea states. When Equation (21) holds, the wave damping terms go to zero and the added inertia terms for $\bar{\sigma} = 0$ may be used. In this case, the ocean surface behaves essentially as a rigid plane,²⁰ and for the case of a buoy whose axis of symmetry is aligned with the y-axis, the added mass in surge is equal to the infinite fluid value. The added mass coefficients for pitch and heave must be calculated separately for each shape considered. Since these coefficients have been studied for several cases of oblate and prolate spheroids,²¹⁻²⁴ and also because they represent mathematical shapes which are similar to surface buoys of sonobuoy systems, it was decided to represent the small buoys by prolate and oblate spheroids. Both types of spheroids are characterized by having two of their three axes equal in length. The limiting cases for a prolate spheroid are a

¹⁸ Newman, J.N., "The Motions of a Spar Buoy in Regular Waves," David Taylor Model Basin Report 1499 (May 1963).

¹⁹ Rudnick, P., "Motion of A Large Spar Buoy in Sea Waves," Journal of Ship Research, Vol. 11, No. 4, pp. 257-267 (Dec 1967).

²⁰ Newman, J.N., "Marine Hydrodynamics (Lecture Notes)," M.I.T. Dept Nav. Arch. and Mar. Eng. (Spring Term 1971).

²¹ Havelock, T., "Waves due to a Floating Sphere Making Periodic Heaving Oscillations," Proceedings of the Royal Society, Vol. 231, Series A, pp. 1-7 (Jul 1955).

²² MacCamy, R.C., "On the Heaving Motion of Cylinders of Shallow Draft," Journal of Ship Research, Vol. 5, No. 3, pp. 34-43 (Dec 1961).

²³ Kim, W.D., "On the Forced Oscillations of Shallow-Draft Ships," Journal of Ship Research, Vol. 7, No. 2, pp. 7-18 (Oct 1963).

²⁴ Kim, W.D., "On a Free-Floating Ship in Waves," Journal of Ship Research, Vol. 10, No. 3, pp. 182-191 (Sep 1966).

long thin cylinder and a sphere, and the limiting cases for an oblate spheroid are a thin circular disk and again a sphere. It can be seen from these limiting cases, that prolate and oblate spheroids can be used to generate a wide range of shapes.

DIFFERENTIAL EQUATIONS OF MOTION

The linearized differential equations for the surge, heave, and pitch motions of a surface buoy or ship freely floating in an inviscid fluid are well known; see, for example, Frank and Salvesen¹³ and Newman.¹⁸ These equations usually contain the inertia forces (including the added hydrodynamic inertia forces), the wave-damping forces, the exciting forces due to the incoming ocean waves, and the restoring forces due to buoyancy. The resulting equations are usually solved in the frequency domain. As mentioned previously, the wave-damping forces may be neglected, to first order, for the two types of buoys considered in the present report. The forces due to viscous drag and cable tension, which are not usually considered in the above studies, are included in the present formulation. The viscous drag forces which are quadratic in the motion velocities of the float, are usually omitted since they make the equations nonlinear and complicate solutions in the frequency domain. The inclusion of these forces poses no problem in a time-domain analysis. The cable forces are, of course, zero for a freely floating buoy.

If the above forces acting on the buoy (shown in Figure 2) are considered and the pitch angle ψ is taken to be small such that

$$\sin \psi \approx \psi \quad (22a)$$

$$\cos \psi \approx 1. \quad (22b)$$

the three differential equations for the surge ξ , heave ζ , and pitch ψ are as follows

$$[A_{\xi\xi} = (m + K_S \rho V)] \ddot{\xi} + \left[A_{\xi\psi} = -\rho \int_0^H (y - y_G) k_S(y) S(y) dy \right] \dot{\psi} = F K_x + D_x + T_x + T_{Iwx} \quad (23)$$

$$[A_{\eta\eta} = (m + K_H \rho V)] \ddot{\zeta} = -\rho g S_w (\zeta - y_w) + F K_y + D_y + T_y - T_{sys} \quad (24)$$

$$A_{\xi\psi} \ddot{\xi} + \left\{ A_{\psi\psi} = \left[1 + \rho \int_0^H (y - y_G)^2 k_S(y) S(y) dy \right] \right\} \dot{\psi} = \rho g V \overline{BG} \dot{\psi} + F K_\psi \\ - \overline{BG} D_x + (-r_y \dot{\psi} + r_x) T_y - (r_x \dot{\psi} + r_y) T_x - r_{wy} T_{Iwx} \quad (25)$$

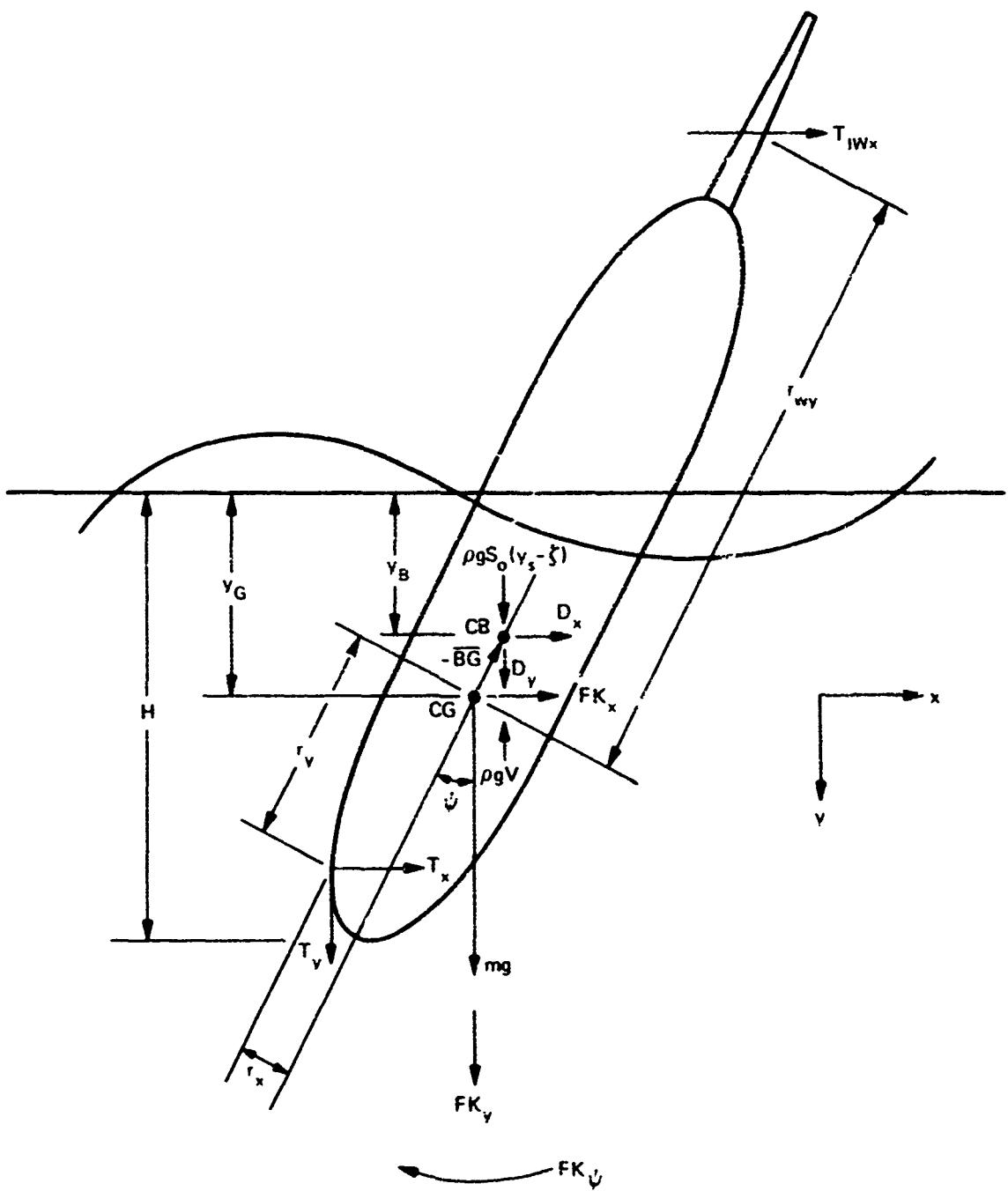


Figure 2 – Definition of Forces Acting on Surface Buoy

where $A_{\xi\xi}$, $A_{\xi\psi}$, $A_{\eta\eta}$, and $A_{\psi\psi}$ = inertia coefficients defined above
 m = mass of the buoy
 K_S and K_H = added mass coefficients for surge and heave, respectively
 V = submerged volume
 H = draft
 y_G = distance of the center of gravity below the undisturbed free surface
 $k_s(y)$ = local added mass coefficient for surge
 $S(y)$ = local cross-sectional area
 FK_x , FK_y = exciting forces due to surface waves in the x- and y-directions, respectively
 FK_ψ = exciting moment about the center of gravity due to surface waves
 D_x , D_y = viscous drag forces in the x- and y-directions, respectively
 T_x , T_y = components of the cable tension in the x- and y-directions, respectively, at the attachment point to the buoy
 T_{Iwx} = wind loading on the buoy in the x-direction
 T_{ys} = steady-state component of tension in the y-direction at the attachment point to the buoy
 S_w = waterplane area of the buoy
 y_w = vertical displacement of the ocean surface, defined in Equation (14b)
 I = moment of inertia about the center of gravity
 $-b\bar{G}$ = $y_B - y_G$
 y_B = distance of the center of buoyancy below the undisturbed free surface
 r_x , r_y = horizontal and vertical distances measured from the center of gravity to the cable attachment point
 r_{wy} = vertical distance measured from the center of gravity to the center of the wind loading force

In Equations (23)–(25), a dot denotes differentiation with respect to time.

The added inertia coefficients, presented below, are all calculated for the case $\dot{\psi} \equiv 0$. It should be noted that the moment terms on the right-hand side of Equation (25) may be readily changed to account for large values of $\dot{\psi}$ which negate approximations (22a) and (22b). However, the added inertia terms on the left-hand side of Equations (23) through (25) must be calculated (by potential flow methods) for each new value of $\dot{\psi}$. For example, for a large value of $\dot{\psi}$, the added mass in surge for a small spheroidal buoy is no longer equal to the infinite fluid value since in this case, there will be motion of the fluid perpendicular to the free surface.

Under static conditions, the submerged volume V must support both the weight of the buoy in air (mg) and the vertical component of the steady-state tension (T_{ys})

$$\rho g V = mg + T_{ys} \quad (26a)$$

$$V = \frac{m}{\rho} + \frac{T_{ys}}{\rho g} \quad (26b)$$

If the input dimensions for the draft and cross-sectional areas of the buoy are such that the submerged volume does not equal the value given in Equation (26b), the program internally multiplies the cross-sectional areas by a constant factor so that the submerged volume becomes exactly equal to the value given by this equation. All of the added inertia and wave-exciting forces given in the following sections are based on this volume.

The following two sections present derivations for the added inertia coefficients K_S , K_H , k_S and the wave-exciting forces FK_x , FK_y , FK_ψ for the two classes of buoys considered: spar buoys and small spheroidal buoys.

SPAR BUOYS

Because of the slenderness of the spar buoy, Newman¹⁸ shows that its added inertias for surge and pitch are identical to those in infinite fluid. Since the spar buoy has a circular cross section, the surge inertia coefficients k_S and K_S are both equal to 1, leading to the following definitions for the added surge and pitch inertia terms in Equations (23) through (25):

$$K_S = k_S = 1 \quad (27a)$$

$$m + K_S \rho V = m + \rho V \quad (27b)$$

$$\rho \int_0^H (y - y_G)^1 k_S(y) S(y) dy = \rho \int_0^H (y - y_G) S(y) dy \quad (27c)$$

$$\rho \int_0^H (y - y_G)^2 k_S(y) S(y) dy = \rho \int_0^H (y - y_G)^2 S(y) dy \quad (27d)$$

Newman takes the buoy to be sufficiently slender so that the added mass for heave may be neglected. Adey and Bai²⁵ have shown experimentally that for the case of a circular cylinder, it is more accurate to add a term corresponding to one-half the added mass of a circular disk (with the same diameter as that of the cylinder) heaving in infinite fluid.

²⁵ Adey, B.H. and K.J. Bai, "Experimental Studies of the Behavior of Spar Type Stable Platforms in Waves," University of California (Berkeley) Report NA-70-4 (Jul 1970).

This correction term has been incorporated into the present formulation by taking the radius of the disk to be the mean radius of the spar buoy \bar{r} , defined by

$$\pi \bar{r}^2 H = V, \bar{r} = \sqrt{\frac{V}{\pi H}}$$

Since one-half of the added inertia of a circular disk heaving in infinite fluid¹¹ is $(4/3) \rho \bar{r}^3$, the following expression is obtained for K_H in Equation (24)

$$K_H = \frac{4}{3} \frac{1}{V} \bar{r}^3 = \frac{4}{3} \frac{1}{V} \left(\frac{V}{\pi H} \right)^{3/2} \quad (28)$$

Newman shows that the wave-exciting forces are simply the Froude-Krylov forces, which may be obtained by integrating the pressure field generated by the ocean waves around the contour of the buoy. The following expressions are obtained for FK_x and FK_ψ

$$FK_x = -2 \sum_{i=1}^N \left[\sigma_i^2 a_{wi} \cos(k_i x - \sigma_i t + \theta_{wi}) Q_0(k_i) \right] \quad (29)$$

$$FK_\psi = 2 \sum_{i=1}^N \left[\sigma_i^2 a_{wi} \cos(k_i x - \sigma_i t + \theta_{wi}) Q_1(k_i) \right] \quad (30)$$

where

$$Q_0(k_i) = \rho \int_0^H e^{-k_i y} S(y) dy$$

$$Q_1(k_i) = \rho \int_0^H e^{-k_i y} (y - y_G) S(y) dy$$

For the heave motion, the Froude-Krylov force given by Newman has been modified to account for the additional heave added-mass term given in Equation (28). This modification has been so made that in the limiting case of a very small spar buoy (which would follow the motions of the waves in the absence of cable forces), the wave-exciting term in Equation (24) would be exactly equal to the inertia term on the left-hand side of the equation. The resulting force has the form

$$FK_y = \left[1 + \frac{4}{3} \rho \left(\frac{V}{\pi H} \right)^{3/2} \right] \sum_{i=1}^N \left[\sigma_i^2 a_{wi} \sin(k_i x - \sigma_i t + \theta_{wi}) Q_0(k_i) \right] \quad (31)$$

The expression given by Newman does not contain the correction term $(4/3) \rho (V/\pi H)^{3/2}$.

SMALL SPHEROIDAL BUOYS

As mentioned previously, for the limiting case of zero reduced frequency where the free surface behaves as a rigid plane, the coefficient for the surge added mass K_S is equal to the infinite fluid value. These coefficients may be calculated by the formulas and tables given in Lamb.¹¹

The added inertia coefficients for heave, pitch, and coupled pitch-surge motions which have components normal to the free surface are not the same as the coefficients for infinite fluid. Instead, they must be calculated separately to incorporate the rigid free-surface condition. Inspection of Equations (23) and (25) shows that the added inertia terms for pitch and coupled pitch-surge motions may be defined as follows.

$$\int_0^H (y - y_G)^2 k_S(y) S(y) dy = \int_0^H (y^2 - 2yy_G + y_G^2) k_S(y) S(y) dy \\ = \mu_{SS} r_w^2 V - 2y_G (\mu_{1S}) r_w V + y_G K_S V \quad (32a)$$

$$- \int_0^H (y - y_G) k_S(y) S(y) dy = \mu_{1S} r_w V + y_G K_S V \quad (32b)$$

where r_w is the maximum radius of the buoy and is used to render the coefficients μ_{SS} and μ_{1S} dimensionless.

Thus, a calculation of μ_{SS} , μ_{1S} , and K_H , along with the values of K_S for infinite fluid as given in Lamb, completely determines all the added inertia terms in Equations (23) through (25). Bai²⁶ has used a finite element approach to calculate coefficients μ_{SS} , μ_{1S} , and K_H for spheroids with draft to maximum radius ratios (H/r_w) ranging from 0.1 to 10. His results agree well with previous results²¹⁻²⁴ for corresponding cases at the zero reduced frequency limit.

Principally for the sake of programming ease, the present program considers only the results for the case where the maximum radius lies at the free surface. Consideration of other radii at the free surface would introduce additional parameters to a description of the submerged buoy. In addition, it is expected that in most cases the maximum radius will be close to the free surface because most of the volume (and hence buoyancy capability) of the spheroid is concentrated in the region around the maximum radius. Results for several cases where the waterline does not occur at the maximum radius are presented in Bai.²⁶

²⁶Bai, K.J., "The Zero-Frequency Hydrodynamic Coefficients of Vertical Axisymmetric Bodies at a Free Surface," Journal of Hydraulics (Jan 1977).

In the range $0.1 \leq H/r_w \leq 10.$, the coefficients μ_{15} , μ_{55} , K_H , and K_S are obtained by linearly interpolating between the values shown in Table 1.

TABLE 1 - VALUES OF ADDED INERTIA COEFFICIENTS
FOR SPHEROIDAL BUOYS AT ZERO REDUCED FREQUENCY

<u>H/r_w</u>	<u>K_S</u>	<u>K_H</u>	<u>μ_{55}</u>	<u>μ_{15}</u>
0.1	0.074	12.84	1.27	0.3
0.2	0.143	5.84	0.55	0.264
0.3	0.207	3.672	0.312	0.237
0.5	0.31	2.005	0.117	0.177
0.7	0.397	1.323	0.0358	0.11
0.9	0.469	0.96	0.0038	0.0039
1.0	0.50	0.836	0.	0.
1.5	0.622	0.484	0.0731	-0.191
2.	0.704	0.323	0.272	-0.391
3.	0.804	0.18	0.993	-0.788
5.	0.894	0.082	3.65	-1.565
7.	0.933	0.049	8.20	-2.38
10.	0.96	0.028	20.0	-3.75

The values of K_S are those given by Lamb,¹¹ and the values of μ_{15} , μ_{55} , and K_H are the values given by Bai,²⁶ subject to two modifications at $H/r_w = 10.0$. The values of μ_{15} and μ_{55} for $H/r_w = 10$, respectively 20.0 and -3.75, correspond to those obtained by simply using strip theory with $k_S(y) = 1$ for $0 \leq y \leq H$. The corresponding values of μ_{15} and μ_{55} obtained by Bai are respectively 17.84 and -3.53. These modifications were made principally for the sake of providing continuity with the approximation used in the range $H/r_w > 10.0$, where the buoy is treated essentially as a spar buoy. There are two reasons for the differences between the strip theory and theoretical finite element calculations.²⁶ First, the strip theory approach neglects the flow around the lower end of the buoy, where $k_S(y) < 1$. Second, the finite element representation, where only the nodes of the elements are on the surface of the buoy, effectively models a smaller buoy. Both of these effects serve to make the strip theory values higher than the corresponding finite element results.

Wave-Exciting Forces

The exciting forces FK_x and FK_y in Equations (23) and (24) were written in a form which assumes that the buoy follows the wave motion in the absence of cable forces and

coupling between pitch and surge motions. This is the case for buoys with dimensions which are small compared to the lengths of the exciting waves. The resulting equations take the form

$$FK_x = (1 + K_S) \rho V \ddot{x}_w = -(1 + K_S) \rho V \sum_{i=1}^N \left[\sigma_i^2 a_{wi} \cos(k_i x - \sigma_i t + \theta_{wi}) \right] \quad (33)$$

$$FK_y = (1 + K_H) \rho V \ddot{y}_w = (1 + K_H) \rho V \sum_{i=1}^N \left[\sigma_i^2 a_{wi} \sin(k_i x - \sigma_i t + \theta_{wi}) \right] \quad (34)$$

The term $e^{-k_i y}$ which appears in Equations (16a) and (16b) for \ddot{x}_w and \ddot{y}_w has been omitted in the above equations since it is ≈ 1 under the assumption of small reduced frequency.

Equation (21).

The pitch-exciting moment is computed by noting that because of the symmetry of the buoy about the vertical axis, only the horizontal wave motions make a contribution to pitch, resulting in

$$\begin{aligned} FK_\psi &= -\rho \int_0^H (y - y_G) \ddot{x}_w [1 + k_S(y)] S(y) dy \\ &= \rho (-\bar{BG} V + \mu_{15} r_w V + y_G K_S V) \ddot{x}_w \\ &= -\rho V (-\bar{BG} + \mu_{15} r_w + y_G K_S) \sum_{i=1}^N \left[\sigma_i^2 a_{wi} \cos(k_i x - \sigma_i t + \theta_{wi}) \right] \end{aligned} \quad (35)$$

Again, under the assumption of small reduced frequency, the term $e^{-k_i y}$ has been set equal to 1 in the expression for \ddot{x}_w .

DYNAMIC CABLE EQUATIONS

GENERAL CONSIDERATIONS

As mentioned previously, equations for the present study are solved in the time domain. Previous cable studies have considered two major approaches in the time domain: the method of characteristics and the finite element method. The method of characteristics is an elegant method which reduces the original set of partial differential equations to a set of ordinary differential equations which are integrated along characteristic lines or wavefronts. This method furnishes valuable insight into the various modes of cable motion but solution times are typically very large.²

The finite element method seeks to represent the actual cable system by a series of segments and nodes. The original set of partial differential equations is then reduced to a set of ordinary differential equations of motion for the nodes. This method facilitates the modeling of nonuniform properties along the cable as well as the presence of intermediate bodies. The method is also quite flexible in that the number and location of nodes is left to the judgment of the user. In arriving at his selection of nodes, he may consider such factors as the type and accuracy of the dynamic information desired, the amount of computer time available, the complexity of the cable system, and the spatial variation of the environmental velocity profiles.

Principally for reasons of generality and flexibility, the finite element approach was used to model the cable. Straight elements are used in the present formulation. Webster²⁷ studied the use of higher order curved elements to model cable shape and concluded that the first order straight element appears to be the most cost-effective. In particular, he showed that one second-order quadratic element would have to be as accurate as at least eight straight elements before its use would be economical.

Before deciding on the final formulation, a number of preliminary approaches for obtaining the differential equations of motion for the nodes were explored. Two approaches in particular were considered in some detail.

PRELIMINARY APPROACHES

In one approach, the equations were formulated in a coordinate system aligned with the cable segment. This is the approach used by Rupe and Thresher²⁸ to obtain the dynamic motions of an inextensible, uniform cable. For the present case of an extensible cable, the two unknowns are the inclination and stretch of each segment. This is the most natural way of describing the configuration of a segment. In addition, certain cable forces such as tension, added inertia, and drag forces are most conveniently expressed in directions normal and tangential to a cable segment. However, in the presence of intermediate bodies along the cable, for which the inertia and drag are most conveniently expressed in the spatial x- and y-directions, the resulting equations are greatly complicated by the transformation required to express the body forces in the cable coordinate system. The cable system considered by Rupe and Thresher is free of intermediate bodies.

²⁷Webster, R.L., "An Application of the Finite Element Method to the Determination of Nonlinear Static and Dynamic Responses of Underwater Cable Structures," General Electric Report R76EMH2 (Jan 1976).

²⁸Rupe, R.C. and Thresher, R.W., "The Anchor-Last Deployment Problem for Inextensible Mooring Lines," ASME Paper 74-WA/Off-T-5 (Dec 1974).

Wang²⁹ has shown that relatively few segments are required to accurately describe the overall steady-state configuration of a cable; these results suggested a second novel approach. The cable was conventionally divided into a number of straight segments and two differential equations in the x- and y-directions were written for the nodes at the ends of the segments. However, each straight segment was subdivided into a number of intermediate nodes, as shown in Figure 3.

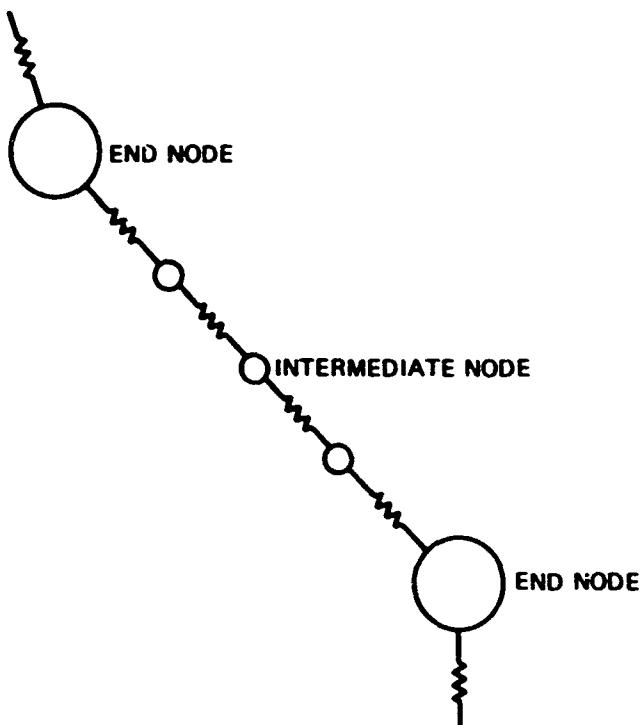


Figure 3 – Finite Element with Intermediate Nodes

Since these intermediate nodes were forced to move along the straight cable segment, only one differential equation was needed to describe their longitudinal motion. The principal intention of this approach was to have the end nodes describe the overall cable configuration and the intermediate nodes describe the variation of tension along a cable segment. There was not sufficient time in the present study to fully explore this approach. However, it was found that a certain amount of bookkeeping was required in the program to differentiate between the "end" and "intermediate" nodes. Also, although this approach reduces the total number of differential equations from that required by more conventional approaches, the

²⁹ Wang, H.T., "Determination of the Accuracy of Segmented Representations of Cable Shape," Journal of Engineering for Industry, Vol. 97, No. 2, Series B, pp. 472-478 (May 1975).

integration time step still depends on the distance between intermediate nodes and the elastic modulus of the cable. For short distances and nearly inextensible cables, the size of the integration steps becomes very small, which increases computer time.

FINAL FORMULATION

In view of the above, selection of the final finite element model was as shown in Figure 4. The continuous cable is divided into a number of massless straight elastic segments. The inertia, weight, and drag forces acting on each cable segment are equally divided between the two nodes at the ends of the segment.

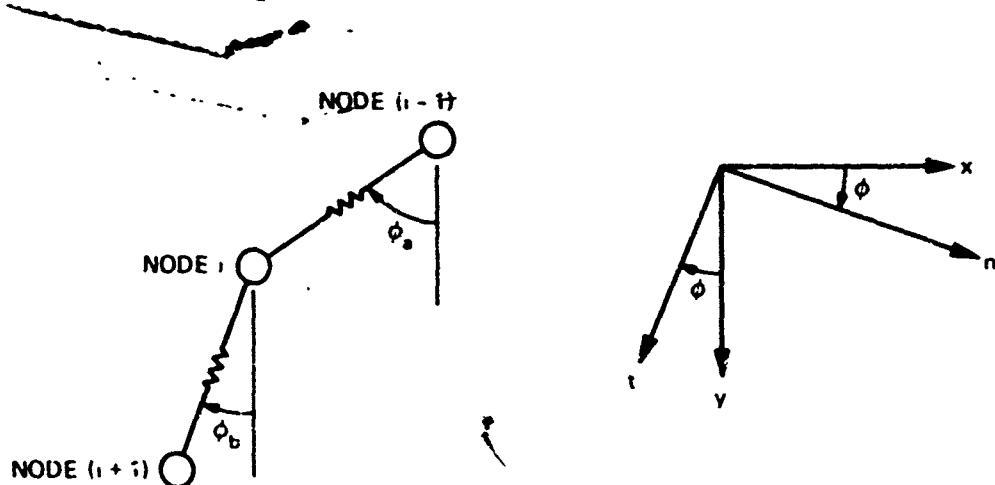


Figure 4 – Final Finite Element Model

Two second-order differential equations of motion are written in the spatial x- and y-directions for each node i , as follows

$$J_{x_i} \ddot{x}_i + K_i \ddot{y}_i = F_{x_i} = T_{x_i} + D_{nx_i} + D_{tx_i} + D_{gx_i} \quad (36)$$

$$K_i \ddot{x}_i + J_{y_i} \ddot{y}_i = F_{y_i} = T_{y_i} + D_{ny_i} + D_{ty_i} + D_{by_i} + W_{C_i} + W_{B_i} \quad (37)$$

$$i = 1, 2, \dots, M$$

where J_x, J_y, K = inertia coefficients defined below in Equation (43)

x, y = subscripts denoting the x- and y-directions, respectively

F = sum of the tension, drag, and gravity forces acting on the node

T = tension in the cable due to stretch and internal damping

D_n = normal drag force acting on the cable

- D_t = tangential drag force acting on the cable
 D_B = drag force acting on the intermediate body
 W_C = weight of the cable in fluid
 W_B = weight of the intermediate body in fluid
 M = total number of nodes

Detailed definitions of the cable forces are now given, and the drag and inertia forces for the intermediate bodies are defined in the next major section.

It is most convenient to solve the set of $2M$ differential equations (36) and (37) with \ddot{x}_i and \ddot{y}_i uncoupled, as follows

$$\ddot{x}_i = (J_{y_i} F_{x_i} - K_i F_{y_i}) / (J_{x_i} J_{y_i} - K_i^2) \quad (38a)$$

$$\ddot{y}_i = (J_{x_i} F_{y_i} - K_i F_{x_i}) / (J_{x_i} J_{y_i} - K_i^2) \quad (38b)$$

DEFINITION OF CABLE FORCES

Inertia Forces

Each cable segment is taken to be a long thin cylinder for which fluid inertia is added only for acceleration normal to the segment. Thus, in a coordinate system aligned with the segment, the inertia force \vec{F}_{IC} for the cable segment is simply given by

$$\vec{F}_{IC} = (\mu \ell_0 + \alpha \rho A \ell_0) \vec{a}_n + \mu \ell_0 \vec{a}_t \quad (39)$$

- where μ = mass per unit length of the cable
 ℓ_0 = reference length of the cable segment
 α = added mass coefficient = 1.0 for a round cable
 A = cross-sectional area of the cable
 \vec{a}_n, \vec{a}_t = accelerations respectively normal and tangential to the cable segment

In this equation, an arrow denotes a vector.

In the fixed x- and y-directions, for which the differential equations are written, the inertia coefficients are not constants but rather are functions of cable inclination ϕ . For the coordinate systems shown in Figure 4, a_n and a_t are related to \ddot{x} and \ddot{y} , respectively the accelerations in the x- and y-directions, by

$$a_n = \ddot{x} \cos \phi - \ddot{y} \sin \phi \quad (40a)$$

$$a_t = -\ddot{x} \sin \phi + \ddot{y} \cos \phi \quad (40b)$$

The x- and y-components of a_n and a_t are, in turn, given by

$$a_{nx} = a_n \cos \phi = \ddot{x} \cos^2 \phi + \ddot{y} \sin \phi \cos \phi \quad (41a)$$

$$a_{ny} = a_n \sin \phi = \ddot{x} \sin \phi \cos \phi + \ddot{y} \sin^2 \phi \quad (41b)$$

$$a_{tx} = -a_t \sin \phi = \ddot{x} \sin^2 \phi - \ddot{y} \sin \phi \cos \phi \quad (41c)$$

$$a_{ty} = a_t \cos \phi = -\ddot{x} \sin \phi \cos \phi + \ddot{y} \cos^2 \phi \quad (41d)$$

The inertia force for the cable segment in the spatial coordinates x and y takes the form

$$\vec{F}_{IC} = [(\mu \ell_0 + \alpha \rho A \ell_0) a_{nx} + \mu \ell_0 a_{tx}] \vec{i} + [(\mu \ell_0 + \alpha \rho A \ell_0) a_{ny} + \mu \ell_0 a_{ty}] \vec{j} \quad (42)$$

where \vec{i} , \vec{j} are unit vectors in the x- and y-directions, respectively.

If one-half of the inertia force of the cable segments above and below the node are summed and the possible presence of an intermediate body at the node is taken into account, Equations (39) to (42) yield the following equation for the total inertia force \vec{F}_I at the node

$$\begin{aligned} \vec{F}_I = & \left[\left(\frac{\mu_a \ell_{oa} + \mu_b \ell_{ob}}{2} + \frac{\alpha \rho A_a \ell_{oa}}{2} \cos^2 \phi_a + \frac{\alpha_b \rho A_b \ell_{ob}}{2} \cos^2 \phi_b + M_{BVx} \right) \ddot{x} \right. \\ & + \left. \left(\frac{-\alpha \rho A_a \ell_{oa}}{2} \sin \phi_a \cos \phi_a - \frac{\alpha \rho A_b \ell_{ob}}{2} \sin \phi_b \cos \phi_b \right) \ddot{y} \right] \vec{i} \\ & + \left[\left(\frac{-\alpha \rho A_a \ell_{oa}}{2} \sin \phi_a \cos \phi_a - \frac{\alpha \rho A_b \ell_{ob}}{2} \sin \phi_b \cos \phi_b \right) \ddot{x} \right. \\ & + \left. \left(\frac{\mu_a \ell_{oa} + \mu_b \ell_{ob}}{2} + \alpha \rho A_a \ell_{oa} \sin^2 \phi_a + \alpha \rho A_b \ell_{ob} \sin^2 \phi_b + M_{BVy} \right) \ddot{y} \right] \vec{j} \\ & = (J_x \ddot{x} + K \ddot{y}) \vec{i} + (K \ddot{x} + J_y \ddot{y}) \vec{j} \end{aligned} \quad (43)$$

where subscripts a and b respectively denote the cable segments above and below the node and M_{BVx} and M_{BVy} are respectively the virtual mass, the mass plus the added mass, of the intermediate body for motions in the x- and y-directions.

Tension Force

For an extensible cable segment, the tension force T depends on the strain ϵ and the strain rate $\dot{\epsilon}$

$$T = T(\epsilon, \dot{\epsilon}) \quad (44)$$

where

$$\epsilon = \frac{\Delta \ell}{\ell_0} = \frac{\ell}{\ell_0} - 1 \approx \frac{\sqrt{(x_\ell - x_u)^2 + (y_\ell - y_u)^2}}{\ell_0} - 1 \quad (45)$$

$$\dot{\epsilon} = \frac{d\epsilon}{dt} = \frac{(\dot{x}_\ell - \dot{x}_u)(x_\ell - x_u) + (\dot{y}_\ell - \dot{y}_u)(y_\ell - y_u)}{\ell \ell_0} \quad (46)$$

Here ℓ is the stretched length of the cable segment and the subscripts ℓ and u respectively refer to the lower and upper ends of the cable segment.

Several relationships have been proposed in the literature for the form of this dependence. In the present study, the following relatively simple yet general relationship is used

$$T = T_0 + C_1 \epsilon^{C_2} + C_1 \dot{\epsilon} \quad (47)$$

where C_1 is the internal damping coefficient. Note that the above dynamic tension differs from the static tension, Equation (8), by the addition of the linear internal structural damping term $C_1 \dot{\epsilon}$. This model, commonly referred to as the Voigt model, has been used in previous cable studies, for example, by Huffman³⁰ and by Goeller and Laura³¹. Goeller and Laura show experimentally that this model adequately describes internal damping for nylon ropes except for frequencies significantly higher than the resonance frequency of the cable. Other more complex forms for the internal structural damping, such as those proposed by Reid,³² can be conveniently incorporated into the program.

Since the cable segments above and below the node act on it, the following two expressions are obtained for T_x and T_y (respectively the x - and y -components of the resultant forces due to cable tension).

$$T_x = T_{ax} + T_{bx} = T_a \sin \phi_a - T_b \sin \phi_b \quad (48a)$$

$$T_y = T_{ay} + T_{by} = -T_a \cos \phi_a + T_b \cos \phi_b \quad (48b)$$

³⁰Huffman, R.R., "The Dynamical Behavior of Extensible Cable in a Uniform Flow Field (An Investigation of the Towed Vehicle Problem)," Ph.D. Thesis, Purdue University (Jan 1969).

³¹Goeller, J.F. and P.A. Laura, "Analytical and Experimental Study of the Dynamic Response of Cable Systems," The Catholic University of America, Department of Mechanical Engineering, Theses Program 893, Report 70-3 (Apr 1970).

³²Reid, R.O., "Dynamics of Deep-Sea Moving Lines," Texas A&M University, Department of Oceanography, AFM Project 204, Reference 68-11F (Jul 1968).

Drag Forces

As shown previously in Equations (6) and (7), the static normal and tangential cable drags are taken to be respectively proportional to the squares of the normal and tangential components of the current velocity \vec{c} . In the dynamic case, the fluid velocity relative to the cable must include the ocean wave particle velocities \dot{x}_w and \dot{y}_w as well as the velocities \dot{x} and \dot{y} of the cable. The resultant expressions v_{rn} and v_{rt} for the relative velocities normal and tangential to the cable then take the form

$$v_{rn} = (c + \dot{x}_w - \dot{x}) \cos \phi + (\dot{y}_w - \dot{y}) \sin \phi = \dot{x}_r \cos \phi + \dot{y}_r \sin \phi \quad (49a)$$

$$v_{rt} = -(c + \dot{x}_w - \dot{x}) \sin \phi + (\dot{y}_w - \dot{y}) \cos \phi = -\dot{x}_r \sin \phi + \dot{y}_r \cos \phi \quad (49b)$$

where $\dot{x}_r = c + \dot{x}_w - \dot{x}$ and $\dot{y}_r = \dot{y}_w - \dot{y}$.

If one-half of the drag forces acting on the cable segments above and below the node are summed, the following equations are obtained for the resultant forces D_{nx} , D_{ny} , D_{tx} , and D_{ty} acting on the node

$$D_{nx} = \frac{1}{2} D_{na} \cos \phi_a + \frac{1}{2} D_{nb} \cos \phi_b \quad (50a)$$

$$D_{ny} = \frac{1}{2} D_{na} \sin \phi_a + \frac{1}{2} D_{nb} \sin \phi_b \quad (50b)$$

$$D_{tx} = -\frac{1}{2} D_{ta} \sin \phi_a - \frac{1}{2} D_{tb} \sin \phi_b \quad (50c)$$

$$D_{ty} = \frac{1}{2} D_{ta} \cos \phi_a + \frac{1}{2} D_{tb} \cos \phi_b \quad (50d)$$

where

$$D_{n(a,b)} = \frac{1}{2} \rho C_D(a,b) d_{(a,b)} \ell_{(a,b)} v_{rn(a,b)} |v_{rn(a,b)}|$$

$$D_{t(a,b)} = \frac{1}{2} \rho C_T(a,b) d_{(a,b)} \ell_{(a,b)} v_{rt(a,b)} |v_{rt(a,b)}|$$

$$v_{rn(a,b)} = \dot{x}_r \cos \phi_{(a,b)} + \dot{y}_r \sin \phi_{(a,b)}$$

$$v_{rt(a,b)} = -\dot{x}_r \sin \phi_{(a,b)} + \dot{y}_r \cos \phi_{(a,b)}$$

INTERMEDIATE BODIES

The magnitude and direction of the added inertia and drag forces for an arbitrary body depend, in general, in a complex manner on body shape and orientation relative to the flow. The modeling of intermediate bodies in previous cable studies has ranged from the very simple to the very complex. At one extreme, the presence of intermediate bodies has been neglected altogether, leading to a cable-only system. At the other extreme, some studies have paid very careful attention to a particular body, often the lower body of the system, and approximated the rest of the cable system in a simple manner.

After careful review of previous studies, formulations were selected for the present study which although relatively simple, can model most bodies of interest for sonobuoy systems. They are also applicable to other cable systems where the bodies are relatively small and/or conform to the shape limitations given below.

CONSTANT COEFFICIENTS

Inertia Forces

The inertia forces in the x- and y-directions are expressed as the virtual mass times the acceleration in these directions, where the virtual mass is the sum of the mass of the body M_B and the constant infinite fluid added mass. Thus, the virtual masses M_{BVx} and M_{BVy} which appear in Equation (43) are given by

$$M_{BVx} = M_B + K_x \rho V_B \quad (51a)$$

$$M_{BVy} = M_B + K_y \rho V_B \quad (51b)$$

where K_x , K_y are respectively the added mass coefficients for motions in the x- and y-directions and V_B is the reference volume, usually the volume of the body.

Drag Forces

Two formulations are used to describe the drag forces. In one, the drag components D_{Bx} and D_{By} are taken to have the form

$$D_{Bx} = \frac{1}{2} \rho C_{DAX} v_r \dot{x}_r \quad (52a)$$

$$D_{By} = \frac{1}{2} \rho C_{DAY} v_r \dot{y}_r \quad (52b)$$

where C_{DAX} , C_{DAY} are respectively the drag areas in the x- and y-directions and $v_r = \sqrt{\dot{x}_r^2 + \dot{y}_r^2}$ is the resultant fluid velocity relative to the body. This approach has been used by Walton and Polacheck³³ and is exact for the case of a sphere. In this case, where $C_{DAX} = C_{DAY} = C_{DA}$, the resultant drag D_B is parallel to the resultant fluid velocity v_r

$$\vec{D}_B = D_{Bx} \vec{i} + D_{By} \vec{j} = \frac{1}{2} \rho C_{DA} v_r (\dot{x}_r \vec{i} + \dot{y}_r \vec{j}) = \frac{1}{2} \rho C_{DA} v_r \vec{v}_r$$

This approach may also be used to approximate the drag for other blunt shapes for which the drag areas for different flow directions do not differ greatly, e.g., near-cubes or circular cylinders with length to diameter ratios of approximately 1.

In the second approach, D_{Bx} and D_{By} are taken as proportional to the squares of the components of the relative fluid velocities in these respective directions

$$D_{Bx} = \frac{1}{2} \rho C_{DAX} \dot{x}_r |\dot{x}_r| \quad (53a)$$

$$D_{By} = \frac{1}{2} \rho C_{DAY} \dot{y}_r |\dot{y}_r| \quad (53b)$$

This is a good approximation for long cylinders or thin disks with axes parallel to the x- or y-directions. In these cases, where there is a large difference between the drag areas, the drag in one direction is essentially pressure drag whereas the much smaller drag in the other direction is essentially due to fluid friction.

In the present program, the choice of whether to use Equation (52) or (53) to compute the drag is determined by the value of the ratio C_{DAX}/C_{DAY} . It was somewhat arbitrarily decided that in the range

$$0.5 \leq C_{DAX}/C_{DAY} \leq 2.0 \quad (54)$$

Equations (52a) and (52b) would be used to compute D_{Bx} and D_{By} . Outside this range, Equations (53a) and (53b) are used to compute D_{Bx} and D_{By} .

VARIABLE COEFFICIENTS FOR CIRCULAR DISK

When a body is executing dynamic oscillations such that it periodically traverses its own viscous wake, the added mass and drag coefficients are more correctly expressed as functions

³³Walton, T.S. and H. Polacheck, "Calculation of Transient Motion of Submerged Cables," Mathematical Tables and Other Aids to Computation, Vol. 14, No. 69, pp. 27-46 (Jan 1960).

of the dynamic motion. Drag coefficients are usually obtained for steady flow and added mass coefficients are often computed for an inviscid fluid. The reviews by Wiegel³⁴ and Holler³⁵ show that nearly all of the measurements of the dynamic coefficients for various bodies have been conducted for dynamic oscillation in one direction only and in the absence of any steady-state fluid velocity. Thus, the results from these studies should be used with caution in the present case where the cable system will generally undergo dynamic motions in both directions in the presence of a steady-state current profile.

For the particular case of a circular disk, which is commonly used in sonobuoy systems to damp out the motions of the lower acoustic units, the user may either employ the constant coefficient approach (described previously) or have the program internally compute the dynamic added mass and drag coefficients for the direction normal to the disk plane, as based on the experimental relationships given by Holler.³⁵ The coefficients are related to the dynamic motions as follows³⁵

$$C_{DAN} = \frac{\pi d_B^2}{4} \frac{2.2}{\gamma} \quad (55a)$$

$$M_{BVn} = M_B + 1.2 \gamma \rho d_B^2 = M_B + M_A \quad (55b)$$

$$\begin{aligned} \gamma &= \sqrt{\beta} = \sqrt{\frac{\ddot{n}^2}{\dot{n} d_B}} \text{ for } 0.077 \leq \beta \leq 3.84 \\ &= \sqrt{3.84} = 1.96 \text{ for } \beta > 3.84 \\ &= \sqrt{0.077} = 0.278 \text{ for } \beta < 0.077 \end{aligned}$$

where n = direction normal to the disk plane, either x or y

M_A = added mass

\dot{n}, \ddot{n} = the velocity and acceleration, respectively, of the disk in this direction, either (\dot{x}, \ddot{x}) or (\dot{y}, \ddot{y})

The above formulas show how the drag area and the added mass vary with β , which is a measure of the ratio of relative magnitudes of the velocity and acceleration. At low values of β , where the acceleration is much higher than the velocity (e.g., during the initial instants of

³⁴ Wiegel, R.L., "Oceanographical Engineering," Prentice Hall, Inc., Englewood Cliffs, N.J., (1964), Chapter 11.

³⁵ Holler, R.A., "Hydrodynamic Effects of Harmonic Acceleration," Naval Air Development Center Report AE-7120 (Jan 1972).

a body starting from rest), the formulas show that C_{DAn} has a high value and that the added mass is given by $M_A = \rho d_B^3/3$, the potential flow result. At the other extreme of high values of β , where the velocity is much higher than the acceleration, the added mass has a high value and the drag area is given by $C_{DAn} = 1.12\pi d_B^2/4$, the steady-state value.

The added mass and drag in the direction tangent to the disk plane, which are much smaller, are computed by the constant coefficient approach outlined previously.

DESCRIPTION OF COMPUTER PROGRAM

Program CABUOY consists of a main program and six subroutines.

MAIN PROGRAM

The main program accepts input data for the cable system, surface waves, current profile, and the initial conditions for the dynamic calculations. If a surface buoy is present, input data are read in by Subroutine BUOY, described below. Data may be entered in either English or metric units. A detailed description of input instructions is given later.

The program is currently written to accept up to 50 cable segments and 49 intermediate bodies. This number can be conveniently increased by changing a few DIMENSION and COMMON statements, but it should be noted that dynamic calculations for more than 50 nodes are likely to require prohibitively large amounts of computer time.

The main program prints out the input data and then calls on various subroutines to calculate the ocean wave spectrum, certain constants for the surface buoy, the steady-state configuration of the cable system, and finally the dynamic motions of the system at prescribed time intervals. The output from the steady-state and dynamic calculations are also printed by this program.

SUBROUTINE STAT

This subroutine defines the five steady-state differential equations, (1) to (5), for the cable.

SUBROUTINE DYN

This subroutine defines the dynamic differential equations, (36) and (37), for each of the M nodes. For cases where a buoy is present, this subroutine also defines the three differential equations of motion for the surface buoy, namely (23) to (25).

SUBROUTINE CUR

This subroutine furnishes the steady-state current profile relative to the cable system. For a free-floating cable system, this would be the actual current profile minus the drift velocity of the cable system. For a given value of the vertical distance y , this subroutine linearly interpolates between the input velocities which are read in as a function of y . For cases where the given value of y is greater (less) than the largest (smallest) value of y which is read in, the subroutine takes the velocity to be the value at the largest (smallest) algebraic value of y which is read in.

SUBROUTINE SPECT

This subroutine is employed when the user wishes the program to internally generate the surface wave components. In this case, the subroutine defines the amplitudes of the surface wave components by using the Pierson-Moskowitz energy sea spectrum, Equation (18). Provision is left at the end of the subroutine for implementing other forms for the sea spectrum.

SUBROUTINE BUOY

This subroutine is used when a surface buoy is present. After reading input data for the surface buoy, the subroutine calculates the various buoy geometrical and added inertia coefficients which appear in Equations (27) to (35). It concludes by calculating the steady-state pitch angle of the buoy, in the absence of any dynamic excitation due to surface waves.

SUBROUTINE ITERA

This subroutine is used for boundary value cases when iteration schemes are required to obtain the steady-state configuration of the cable system. It contains iteration schemes which are applicable for free-floating cable systems and a cable of given length moored in a given ocean depth. Provision is made at the end of the subroutine for implementing iteration schemes for other applications.

SUBROUTINE KUTMER

This subroutine uses the Kutta-Merson method to numerically integrate the steady-state differential equations defined in Subroutine STAT and the dynamic differential equations of

motion defined in Subroutine DYNA. The subroutine automatically reduces the integration step size until specified error criteria are met.

INPUT INSTRUCTIONS

READ STATEMENTS

Input data are entered into the program by means of the following READ statements contained in Program MAIN and Subroutine BUOY. These statements are given numbers simply for identification purposes.

MAIN Program

READ (5,1) NCASES	Card 1
DO 1000 MC= 1, NCASES	
READ (5,301) TITLE	Card 2
READ (5,1) NSM, NSW, NCAB, NCUR, ITER, MTRC	Card 3
READ (5,2) (FSM (K), K = 1, NSM)	Card 4
READ (5,2) (AXSM (K), K = 1, NSM)	Card 5
READ (5,2) (AYSM (K), K = 1, NSM)	Card 6
READ (5,2) (FIDSM (K), K = 1, NSM)	Card 7
READ (5,2) (ASW (K), K = 1, NSW)	Card 8
READ (5,2) (FRSW (K), K = 1, NSW)	Card 9
READ (5,2) (FIDSW (K), K = 1, NSW)	Card 10
READ (5,2) RHO, SUBM, TWX, TIY, CDASX, AMC, AFAC, TMIN	Card 11
READ (5,2) TINV1, DT1, TOTT, DT2, DIR, TBH, TBYMX	Card 12
READ (5,3) (FLC (K), K = 1, NCAB)	Card 13
READ (5,2) (DC1 (K), K = 1, NCAB)	Card 14
READ (5,2) (CDN (K), K = 1, NCAB)	Card 15
READ (5,2) (CDT (K), K = 1, NCAB)	Card 16
READ (5,2) (WC (K), K = 1, NCAB)	Card 17
READ (5,4) (CM (K), K = 1, NCAB)	Card 18
READ (5,3) (TREF (K), K = 1, NCAB)	Card 19
READ (5,5) (C1 (K), K = 1, NCAB)	Card 20
READ (5,2) (C2 (K), K = 1, NCAB)	Card 21
READ (5,2) (CINT (K), K = 1, NCAB)	Card 22
READ (5,2) (WBD (K), K = 1, NCAB)	Card 23
READ (5,2) (CDABX (K), K = 1, NCAB)	Card 24
READ (5,2) (CDABY (K), K = 1, NCAB)	Card 25
READ (5,2) (XMBV (K), K = 1, NCAB)	Card 26
READ (5,2) (YMBV (K), K = 1, NCAB)	Card 27
READ (5,3) (YY (I), I = 1, NCUR)	Card 28
READ (5,3) (CCK (I), I = 1, NCUR)	Card 29
READ (5,2) (PHID (I), I = 1, NCAB)	Card 30

READ (5,3) (TENI(I), I=1, NCAB)	Card 31
READ (5,2) (XPI(I), I=1, NCAB)	Card 32
READ (5,2) (YPI(I), I=1, NCAB)	Card 33

1000 CONTINUE

The corresponding FORMAT statements are:

```

1 FORMAT (24I3)
2 FORMAT (8F10.4)
3 FORMAT (8F10.2)
4 FORMAT (8F10.6)
5 FORMAT (8F10.0)
301 FORMAT (20A4)

```

Subroutine BUOY

READ (5,1) CIASY, WAS, RWY, RTX, RTY, YCG, BIN	Card 34
READ (5,1) XSI, ZETI, SYDI, XPSI, ZTPSI, SYPDI	Card 35

The corresponding FORMAT statement is:

```
1 FORMAT (8F10.4)
```

DEFINITION OF INPUT VARIABLES FOR MAIN PROGRAM

NCASES	Number of cases, NCASES ≥ 1
TITLE	Title
NSM ²	Number of surface motion components, 1 \leq NSM \leq 20
NSW ³	Number of surface wave components, 1 \leq NSW \leq 20
NCAB	Number of cable segments, 2 \leq NCAB \leq 50
NCUR	Number of current profile points, 2 \leq NCUR \leq 10
MTRC	MTRC ≤ 0 if input data are entered in English units; MTRC ≥ 1 if input data are entered in metric units
ITER	Iteration index
FSM(K) ²	
AXSM(K) ²	$x_{SM} = \sum_{k=1}^{NSM} AXSM(K) * \cos(-2\pi * FSM(K) * t + FIDSM(K) * \pi / 180.)$
AYSM(K) ²	$y_{SM} = \sum_{k=1}^{NSM} -AYSM(K) * \sin(-2\pi * FSM(K) * t + FIDSM(K) * \pi / 180.)$
FIDSM(K) ²	
ASW(K) ³	$x_{SW} = \sum_{k=1}^{NSW} ASW(K) * \cos(-2\pi * FRSW(K) * t + FIDSW(K) * \pi / 180.)$

FRSW(K)³	
FIDSW(K)³	$y_{SW} = \sum_{k=1}^{NSW} -ASW(k) * \sin(-2\pi^* FRSW(k) * t + FIDSW(k) * \pi / 180.)$
RHO	Fluid density in slugs/feet ³ (kilograms/meters ³)
SUBM⁴	Submergence of top point of cable below free surface in feet (meters)
TWX⁴	Horizontal force acting at top of cable in pounds (newtons)
TIY	Vertical component of tension at top of cable in pounds (newtons)
CDASX⁴	Drag area of surface buoy perpendicular to the x-axis in feet ² (meters ²)
AMC	Added mass coefficient of cable; AMC = 1.0 for round cable
AFAC	Cross-sectional area of cable = AFAC * $\pi d^2 / 4$; AFAC = 1.0 for round cable
TMIN	Minimum algebraic tension which can be supported by cable in pounds (newtons)
TINV1	Initial time interval in seconds for dynamic calculations
DT1	Time step in seconds for which printout is desired for $0 \leq t \leq TINV1$
TOTT	Total time in seconds for which dynamic calculations are desired
DT2	Time step in seconds for which printout is desired for $TINV1 < t \leq TOTT$
DIR	$DIR < 0$, if initial conditions are prescribed at the bottom (towing cable case); otherwise $DIR \geq 0$
TBH	Applied force in pounds (newtons) on lower weight, body NCAB-i, in x-direction
TBYMX	Maximum absolute value in pounds (newtons) of tension in cable just below buoy; for buoy-cable system, set TBYMX equal to a large number, say, 99999
FLC(K)	Length of Kth cable segment in feet (meters)
DCI(K)	Diameter of Kth cable segment in inches (centimeters)
CDN(K)	Normal drag coefficient of Kth cable segment
CDT(K)	Tangential drag coefficient of Kth cable segment
WC(K)	Weight in fluid in pounds/foot (newtons/meter) of Kth cable segment at the reference cable tension
CM(K)	Mass of Kth cable segment in slugs/foot (kilograms/meter) at the reference cable tension
TREF(K)	Reference tension in pounds (newtons) of Kth cable segment
C1(K)⁵, C2(K)	Tension = $TREF(K) + C1(K) * \epsilon^{C2(K)} + CINT(K) * \dot{\epsilon}$; for linearly elastic material, $C1(K) = AE$ and $C2(K) = 1$
CINT(K)	
WBD(K)	Weight in fluid of Kth body in pounds (newtons)
CDABX(K)⁶	Drag area of Kth body in feet ² (meters ²) for flow in (x,y) directions
CDABY(K)⁶	
XMBV(K)⁶	Virtual mass (mass + added mass) in slugs (kilograms) of Kth body in (x,y) directions
YMBV(K)⁶	

YY(I)⁷	Value of y in feet (meters)
CCK(I)	Value of current in knots (meters/second) at y = YY(I)
PHID(I)⁸	Initial value of ϕ of Ith cable segment in degrees
TENI(I)⁸	Initial value of tension of Ith cable segment in pounds (newtons)
XPI(I)	Initial value of \dot{x} of Ith node in feet/second (meters/second)
YPI(I)	Initial value of \dot{y} of Ith node in feet/second (meters/second)

DEFINITION OF INPUT VARIABLES FOR SURFACE BUOY

CDASY	Drag area for y-direction in feet ² (meters ²)
WAS	Weight in air in pounds (newtons)
RWY	Vertical distance of wind loading center of pressure from buoy center of gravity YCG in feet (meters)
RTX, RTY	(x, y) distance of cable attachment point from YCG in feet (meters)
YCG	Submergence of center of gravity below the free surface under the action of its own weight WAS and the vertical component of the steady-state tension (-TIY) in feet (meters)
BIN	Moment of inertia in air about YCG in slug feet ² (kilogram meters ²)
XSI, ZETI,	Initial values of (x, ξ , ψ) in (feet, feet, degrees)(meters, meters, degrees), where ξ is the vertical displacement of the center of gravity from its equilibrium value YCG
SYDI⁹	
XPSI, ZTPI,	Initial values of (\dot{x} , $\dot{\xi}$, $\dot{\psi}$) in (feet/second, feet/second, degrees/second)
SYPDI	(meters/second, meters/second, degrees/second)

EXPLANATORY NOTES

1. **ITER = 0**, no iteration (prescribed initial steady-state conditions)
 1. free-floating cable system
 2. moored cable with given length in given depth
 3. iteration scheme to be programmed by user
2. For $1000 \leq FSM(1) < 2000$, the program makes the prescribed surface motion components equal to the surface wave components by setting $AXSM(K) = AYSM(K) = ASW(K)$, $FSM(K) = FRSW(K)$, and $FIDSM(K) = FIDSW(K)$ for $K = 1$ to $K = NSM$; the program automatically sets $NSM = NSW$.

For $2000 \leq FSM(1) < 3000$, the program accepts input data for a spar buoy and considers $AXSM(K)$ to be the cross-sectional area of the buoy in feet² (meters²) at depth $AYSM(K)$ feet (meters) below the free surface. $AYSM(1) = 0$, and $AYSM(NSM) =$ total draft under the combined action of buoy weight in air and the vertical component of the

steady-state tension. NSM should be an odd number. The input values for FIDSM(K) may take on any values such as, say, 0.

For $FSM(1) > 3000$, the program accepts input data for a spheroidal buoy and considers AXSM(1) to be the radius of the buoy cross section at the free surface and AYSM(1) to be the total draft. The rest of the input values of AXSM(K) and AYSM(K) as well as all of the FIDSM(K) may take on any values, e.g., 0.

3. For $ASW(1) > 1000$, the program computes the amplitude of the ASW surface wave components by using the Pierson-Moskowitz sea spectrum. In these cases, the program considers the significant wave height in feet (meters) to be $(ASW(1) - 1000)$ and FRSW(1) and FRSW(2) to respectively be the lower and upper frequencies of the spectrum in cycles per second. The program internally generates the phases of the wave components by considering them to be uniformly separated by $360/NSW$ degrees. The phase of the lowest frequency component, in degrees, is taken to be the input value of FIDSW(1).

4. For the case of a surface buoy ($FSM(1) \geq 2000$), the program calculates the drag acting on the surface buoy due to the ocean current by taking the value of the ocean current SUBM feet (meters) below the free surface. Thus, $0 \leq SUBM \leq$ total draft.

The total horizontal force at the top point of the cable $TIX = TWX + (1/2)\rho * CDASX * CCF(SUBM) * ABS(CCF(SUBM))$. In cases where there is no surface buoy (i.e., prescribed surface motion), TWX and/or CDASX may be set equal to zero. For cases of a surface buoy, TWX represents the wind loading on the buoy in pounds (newtons).

5. For free-floating and towing cables where the last ($K = NCAB$) cable connecting the lower weight to the ocean bottom is fictitious, read in a value for $C1(NCAB)$ less than 0.0001 pounds (0.0004 newtons). In these cases, the program sets DCI(NCAB) = CDN(NCAB) = CDT(NCAB) = WC(NCAB) = CM(NCAB) = CINT(NCAB) = 0, FLC(NCAB) = 2 * FLC(NCAB - 1), and C2(NCAB) = 1.

6. If CDABX(K) is negative, the program considers the body to be a circular disk with plane perpendicular to the x-axis and calculates drag and added mass forces by using the formulation given in Equation (55). In these cases, CDABX(K) is the negative of the actual drag area and XMBV(K) is the mass (not the virtual mass) of the disk. In these cases, CDABY(K) and YMBV(K) should be positive and retain the definitions given previously. Similar remarks apply if CDABY(K) is read in as a negative number except that the plane of the disk is now perpendicular to the y-axis.

7. When ITER = 2, the program takes YY(NCUR) to be the ocean depth.

8. For $|PHID(1)| \geq 360$, the program takes the initial values of the angle and tension of each cable segment to correspond to their respective steady-state values at the midpoint of

each segment. These steady-state values have been previously calculated by the program. This approach will minimize transient dynamic effects. In these cases, input values for the remaining PHID(K) as well as all of the TENI(K) may be arbitrary, e.g. 0.

9. For SYDI \geq 360., the program sets the initial value for buoy inclination ψ equal to the steady-state value of ψ , which has previously been calculated by the program. This will tend to minimize transient dynamic motions of the surface body.

PROGRAM STORAGE AND TIME REQUIREMENTS

On the CDC 6700 currently in use at the Center, the program requires a memory of approximately 47,200 octal words to load and 33,700 octal words to execute. Compilation time is approximately 23 seconds. Execution time for a particular case depends on a large number of factors, the most important of which include the tension-strain relation of the cable, the number of cable nodes, the frequencies of the exciting surface waves and the prescribed surface motion (if any), and the amount of time over which the dynamic motions are desired. Table 2 shows the computer execution time ET and cost for all of the sample problems presented in the following chapter. A computer priority (CP) of P2 indicates overnight priority whereas P3 is the standard daytime priority at the DTNSRDC Computer Center.

**TABLE 2 – COMPUTER EXECUTION TIMES AND COST
FOR ALL THE SAMPLE PROBLEMS**

<u>Prob.</u>	<u>NCAB-1</u>	<u>C₁ lb (4.45N)</u>	<u>C₂</u>	<u>ET sec</u>	<u>CP</u>	<u>Total Cost \$</u>
1A	4	2.4×10^1	1.0	61.7	P2	9.93
1B	4	2.4×10^2	1.0	57.8	P2	9.56
1C	4	2.4×10^3	1.0	62.4	P2	9.99
1D	4	2.4×10^4	1.0	85.0	P2	12.00
1E	4	2.4×10^5	1.0	250.0*	P2	28.00*
1F	4	2.4×10^3	0.5	1000.0*	P2	100.00*
1G	4	2.4×10^3	2.0	55.2	P2	8.93
2A	1	2.4×10^3	1.0	9.3	P2	5.15
2B	2	2.4×10^3	1.0	25.1	P2	6.60
2C	4	2.4×10^3	1.0	62.4	P2	9.99
2D	8	2.4×10^3	1.0	180.0*	P2	20.00*
2E	15	2.4×10^3	1.0	600.0*	P2	60.00*
3A	4	2.0×10^1	1.0	90.0**	P3	15.00**
3B	4	2.0×10^1	1.0	100.0**	P3	16.00**
3C	4	2.0×10^1	1.0	100.0**	P3	16.00**
4A	4	****	****	82.6	P3	14.27
4B	4	****	****	74.2	P2	11.10
4C***	4	****	****	100.0*	P3	16.00*

* Extrapolated to 50 sec of dynamic motion

** Extrapolated to 20 sec of deployment time

*** Six surface wave components

**** See Figure 11

SAMPLE PROBLEMS

Input cards are listed for the four sample problems presented to illustrate use of the program. Representative portions of the program output are listed for one of the cases of Problem 1, but only some final results are shown for the other three problems.

PROBLEM 1 – UNIFORM CABLE WITH VARIOUS TENSION-STRAIN RELATIONS

Problem: Compute the dynamic behavior of a cable suspended from an ocean platform in the presence of a uniform current of 1 knot in the +x-direction for different values of the elastic constants C_1 and C_2 appearing in Equation (47)

Case	C1 lb (4.45 N)	C2
A	2.4×10	1.0
B	2.4×10^2	1.0
C	2.4×10^3	1.0
D	2.4×10^4	1.0
E	2.4×10^5	1.0
F	2.4×10^3	0.5
G	2.4×10^3	2.0

The fixed surface motion, surface wave, cable, and lower body parameters are as follows:

Surface motion = surface wave:

number of components	1
frequency	0.1 cps
amplitude in x- and y-directions	10 ft (3.05 m)
phase angle	0 deg

Cable:

length	1000 ft (305 m)
diameter	0.2 in. (0.508 cm)
normal drag coefficient	1.4
tangential drag coefficient	0.01
weight in fluid	0.01 lb/ft (0.146 N/m)
mass	0.001 slugs/ft (0.0478 kg/m)
reference tension	25 lb (111.2 N)
internal damping coefficient	0

Lower weight:

weight in fluid	20 lb (89 N)
drag area in x-direction	0.3 ft ² (0.0279 m ²)
drag area in y-direction	0.3 ft ² (0.0279 m ²)
virtual mass in x-direction	1.0 slugs (14.6 kg)
virtual mass in y-direction	1.0 slugs (14.6 kg)

Fluid density:

1.94 slugs/ft³ (1000.6 kg/m³)

Represent the cable by four equal segments, not including the fictitious cable segment below the lower weight. Dynamic motions are desired for a total of five cycles of the surface motions, i.e. 50 sec. For the initial interval of 10 sec. print out the transient motions every

0.25 sec. For the final 40 sec, increase the printout interval to 1.0 sec. Let the cable start from rest with the initial angle and tension of each segment equal to the steady-state values.

Solution: The data cards for this problem are listed in Table 3. The cards which are the same for all the cases are listed at the top of the table. The cards for the title and the elastic constants C1 and C2, which differ for each case, are listed at the bottom of the table. The symbol b denotes a blank in this and subsequent tables which list data cards for the sample problems. Also, Column 1, 11, 21, 31, 41, 51, 61, and 71 have been indicated since most of the data start in these columns.

TABLE 3 – INPUT DATA FOR SAMPLE PROBLEM 1

	1	11	21	31	41	51	61	71
Card 1	bb1							
Card 3	bb1bb1bb5bb2bb0							
Card 4	0.1							
Card 5	10.0							
Card 6	10.0							
Card 7	0.							
Card 8	10.0							
Card 9	0.1							
Card 10	0.							
Card 11	1.94	0.	0.	-50.	0.	1.0	1.0	0.
Card 12	10.	0.25	50.	1.0	-1.0	0.	99999.	
Card 13	250.	250.	250.	250.				
Card 14	0.2	0.2	0.2	0.2				
Card 15	1.4	1.4	1.4	1.4				
Card 16	0.02	0.02	0.02	0.02				
Card 17	0.01	0.01	0.01	0.01				
Card 18	0.001	0.001	0.001	0.001				
Card 19	25.	25.	25.	25.				
Card 22	0.	0.	0.	0.				
Card 23	0.	0.	0.	20.				
Card 24	0.	0.	0.	0.3				
Card 25	0.	0.	0.	0.3				
Card 26	0.	0.	0.	1.0				
Card 27	0.	0.	0.	1.0				
Card 28	0.	10000.						
Card 29	1.	1.						

TABLE 3 – (continued)

	1	11	21	31	41	51	61	71
Card 30	999.	0.	0.	0.				
Card 31	0.	0.	0.	0.				
Card 32	0.	0	0.	0.				
Card 33	0.	0.	0.	0.				
Card 2	bbb PROBLEM 1A, C1 = 24, C2 = 1							
Card 20	24.	24.	24.	24.				
Card 21	1	1.	1.	1.				
Card 2	bbb PROBLEM 1B, C1 = 240, C2 = 1							
Card 20	240.	240.	240.	240.				
Card 21	1.	1.	1.	1.				
Card 2	bbb PROBLEM 1C, C1 = 2400, C2 = 1							
Card 20	2400.	2400.	2400.	2400.				
Card 21	1.	1.	1.	1.				
Card 2	bbb PROBLEM 1D, C1 = 24000, C2 = 1							
Card 20	24000.	24000.	24000.	24000.				
Card 21	1.	1.	1.	1.				
Card 2	bbb PROBLEM 1E, C1 = 240000, C2 = 1							
Card 20	240000.	240000.	240000.	240000.				
Card 21	1.	1.	1.	1.				
Card 2	bbb PROBLEM 1F, C1 = 2400, C2 = 0.5							
Card 20	2400.	2400.	2400.	2400.				
Card 21	0.5	0.5	0.5	0.5				
Card 2	bbb PROBLEM 1G, C1 = 2400, C2 = 2							
Card 20	2400.	2400.	2400.	2400.				
Card 21	2.0	2.0	2.0	2.0				

Table 4 shows the first six and the last two pages of the computer output for Problem 1D. The pages which have been left out simply contain output for the dynamic motions for intermediate time intervals. Table 4 shows that the first page of the output lists a table of conversion from English to metric units. The second page lists the input data. For the present case of a suspended cable, where the initial conditions are known at the lower body, two integrations are performed for the ste. γ -state configuration (see section on Initial Value Cases). The results of these two integrations are given in the next two pages. The remainder of the output is a listing of the dynamic displacements, velocities, and accelerations of the surface waves, upper cable point, and each node at the prescribed time intervals. The output also lists the angle, angular velocity, tension, strain, and strain rate of each cable segment.

TABLE 4 - PROGRAM OUTPUT FOR PROBLEM 1D

CONVERSION FROM ENGLISH UNITS TO METRIC UNITS

ENGLISH	METRIC
1 INCH=	2.540 CM
1 FOOT=	0.3048 METERS
1 SQ FT=	0.09290 SQ M
1 CU FT=	0.02831 CU M
1 POUND=	4.452 NEWTONS
1 SLUG=	14.607 KGM
1 SECOND=	1 SECOND
1 KNOT=	0.5144M/SEC

TABLE 4 -- (continued)

PROBLEM 1D.C1=25000.C2=1									
LISTING OF ENVIRONMENTAL AND CABLE-BUOY CHARACTERISTICS									
OCEAN CONDITIONS									
SURFACE WAVE-	FREQUENCY	AMPL(FT)	PHASE(D)	WL(FT)	WK(1/FT)				
.10	10.00	0.00	0.00	512.44	0.0123				
CURRENT PROFILE	DEPTH(FT)	CURRENT							
	0.00	1.00							
	10000.00	1.00							
FLUID DENSITY	1.9400 SL/CFUFT								
SURFACE MOTION-FREQ(CPS)	X-A(FT)	Y-A(FT)	Z-A(FT)	PHASE(DEG)					
.1000	10.0000	10.0000	10.0000	0.0000					
GENERAL CABLE CHARACTERISTICS									
AN EQUPT AREA FAC T MIN(LB)									
1.0000	1.0000	1.0000	1.0000	1.0000					
CABLE PROPERTIES									
NUM	LEN(FT)	DIA(INCH)	CON	CDT	W(LB/FT)	T(REF(LB))	C1(LB)	C2(LB)	BODY PROPERTIES
1	250.00	.2000	1.4000	.0200	.00100	25.00	24000.	1.0000	XWM(SL)
2	250.00	.2000	1.4000	.0200	.00100	25.00	24000.	0.9999	YWM(SL)
3	250.00	.2000	1.4000	.0200	.00100	25.00	24000.	0.9999	
4	250.00	.2000	1.4000	.0200	.00100	25.00	24000.	0.9999	
5	250.00	.2000	1.4000	.0200	.00100	25.00	24000.	0.9999	
K-LOAD ON BOT WT.	0.0000 LB								
SURFACE BUOY CHARACTERISTICS									
SUBM(FT)	MINW LD(LB)	COAX(F1SQ)	MAX TENS(LB)						
0.0000	0.0000	0.0000	99999.0000						
INITIAL VALUES AT TOP OF CABLE									
TA= 10.0000	YA= 0.0000	XVA= 0.0000	YVA= 0.0000						

TABLE 4 - (continued)

RUN NUMBER 1					
STEADY-STATE CONFIGURATION					
	X(FT)=	Y(FT)=	Z(FT)=	DIRECTION=	PHIS (DEG)=
MODE S	REF(FT)	S STRFTI	X(FT)	Y(FT)	Z(FT)
0	3.03	0.33	10.00	0.00	23.82
4	125.00	126.96	-17.97	-121.13	21.24
4	250.33	249.96	-87.72	-226.86	22.34
4	250.00	249.96	-80.72	-228.86	22.34
5	375.00	374.95	-163.82	-322.84	23.33
5	500.00	499.96	-258.39	-403.68	24.21
2	500.00	499.96	-258.39	-403.68	24.21
2	625.00	624.94	-359.72	-476.83	25.02
2	750.00	749.94	-465.24	-543.82	25.77
1	750.00	749.94	-465.24	-543.82	25.77
1	875.00	874.95	-573.50	-686.31	26.48
1	1000.00	999.96	-663.63	-665.45	27.16
0	1000.00	999.96	-663.63	-665.45	27.16
					117.52
X-COMPONENT OF TENSION=					
Y-COMPONENT OF TENSION=					
REVISED VALUE OF WIND LD=					
INITIAL VALUES AT TOP OF CABLE					
XA=	10.333	YA=	0.3000	XVA=	0.3300
				VVA=	6.2832

TABLE 4 - (continued)

RUN NUMBER 1					
STEADY-STATE CONFIGURATION					
XIX=	-24.09	YIV=	-12.55	DIRECTION=	1.00
NODE S	REF(FT)	S STR(FT)	X(FT)	Y(FT)	TEN(LB)
9	3.33	0.33	10.33	0.03	27.16
1	129.00	125.01	120.13	59.14	-62.18
1	250.00	250.02	226.39	121.63	26.48
1	250.00	250.02	226.39	121.63	25.77
2	375.00	375.02	333.91	188.62	25.32
2	500.00	500.02	435.24	261.77	-52.91
2	500.00	500.02	435.24	261.77	24.21
3	625.00	625.01	529.81	363.41	-45.98
3	750.00	750.01	612.91	436.60	22.34
3	750.00	750.01	612.91	436.60	22.34
4	875.00	874.98	675.66	546.32	-22.66
4	1000.00	999.96	703.63	665.45	20.02
4	1000.00	999.96	703.63	665.45	-2.37
5	1250.00	1279.45	424.63	648.81	93.41
5	1500.00	1558.95	145.63	632.17	93.41
5	1500.00	1558.95	145.63	632.17	93.41
X-COMPONENT OF TENSION= -0.00 LB					
Y-COMPONENT OF TENSION= -0.00 LB					
CABLE INITIAL CONDITIONS					
NODE	PHI(0deg)	TEN(LB)	X(FT)	Y(FT)	XV(FT/S)
1	-63.9757	26.48	228.62	121.30	0.0000
2	-56.0634	25.02	436.08	260.00	0.0000
3	-45.9844	23.33	615.85	436.50	0.0003
4	-22.6616	21.24	712.16	665.16	0.0000
5	93.4133	.03	154.16	631.66	0.0000
TIME INFORMATION					
INITIAL TIME INTERVAL=	10.0000 SEC	TIME STEP=	.2500 SEC		
TOTAL TIME=	50.0000 SEC	TIME STEP=	1.0000 SEC		

TABLE 4 – (continued)

COMPUTED CABLE SYSTEM MOTIONS AND TENSIONS

NUM	T (SEC)	DT (SEC)	X (FT)	Y (FT)	XP(F/T/S)	YP(F/T/S)	XPP(F/T/S)	YPP(F/T/S)	TEN(LB)		FIP(0/S)	STRAIN STR(1,1)
									XPP(F/S)	YPP(F/S)		
WAVE												
0	-2.5000	.0032500	9.50	1.56	-1.9029	6.2298	-3.6932	-6.6176	17.43	-61.38	-1.78	-0.00315 - 0.001950
1	-2.5000	.0032500	9.50	1.56	-1.9029	6.2298	-2.0510	-11.1273	-6.6133	18.45	-55.99	-0.43 - 0.003500 - 0.01596
2	-2.5000	.0032500	229.25	2.7698	-1.7939	11.1273	-2.5335	-10.2659	-5.98	-45.98	-0.97 - 0.00369 - 0.00274	
3	-2.5000	.0032500	436.36	2.6076	-1.6055	11.4332	-2.5335	-11.1653	6.75	-22.64	-0.23 - 0.00364 - 0.00274	
4	-2.5000	.0032500	615.99	436.61	1.7031	1.7346	6.7332	-1.1653	-1.1653	6.75	-22.64	-0.23 - 0.00364 - 0.00274
5	-2.5000	.0032500	615.99	436.61	1.7031	1.7346	6.7332	-1.1653	-1.1653	6.75	-22.64	-0.23 - 0.00364 - 0.00274
6	-2.5000	.0032500	615.99	436.61	1.7031	1.7346	6.7332	-1.1653	-1.1653	6.75	-22.64	-0.23 - 0.00364 - 0.00274
7	-2.5000	.0032500	712.16	615.16	1.8053	5.5735	5.9886	12.2671	93.61	-93.61	-0.96	-117.973 - 0.00364
WAVE												
0	-5.0000	.001953	9.51	3.89	-1.9016	5.9737	-3.7586	-1.2288	11.24	-61.81	-1.76	-0.00373 - 0.001117
1	-5.0000	.001953	229.73	121.12	-1.7537	5.7665	9.1461	5.7486	13.45	-55.99	-0.42 - 0.00361 - 0.00339	
2	-5.0000	.001953	436.62	201.26	-1.6599	1.3220	17.6853	15.9883	18.62	-45.98	-0.12 - 0.00329 - 0.003242	
3	-5.0000	.001953	616.35	436.93	1.6106	2.2326	7.7311	7.1857	19.21	-22.59	-0.19 - 0.00321 - 0.00321	
4	-5.0000	.001953	712.36	615.69	1.8053	2.4651	5.4355	-11.3346	93.67	-93.67	-0.27 - 0.00343 - 0.002499	
WAVE												
0	-7.5000	.001953	9.51	4.91	-1.9016	5.5916	-3.5176	-1.7923	49.71	-62.25	-1.76	-0.00355 - 0.00226
1	-7.5000	.001953	229.73	121.12	-1.7537	5.5925	5.5916	-1.65216	32.31	-55.74	-0.37 - 0.00375 - 0.003235	
2	-7.5000	.001953	436.62	201.26	-1.6599	1.3220	-1.65216	-10.6335	24.52	-45.95	-0.97 - 0.00329 - 0.003242	
3	-7.5000	.001953	616.35	436.93	1.6106	2.2326	-1.65216	-1.6335	26.86	-22.59	-0.12 - 0.00321 - 0.00321	
4	-7.5000	.001953	712.36	615.69	1.8053	2.4651	5.4355	-11.3346	93.67	-93.67	-0.27 - 0.00343 - 0.002499	
WAVE												
0	-7.5000	.015625	8.91	4.91	-2.0525	5.5916	-3.5176	-1.7923	49.71	-62.25	-1.76	-0.00355 - 0.00226
1	-7.5000	.015625	238.38	121.83	-1.6599	1.3220	-1.65216	-1.6335	24.52	-45.95	-0.97 - 0.00329 - 0.003242	
2	-7.5000	.015625	437.88	261.67	1.6106	2.2326	-1.65216	-1.6335	26.86	-22.59	-0.12 - 0.00321 - 0.00321	
3	-7.5000	.015625	616.75	436.93	1.6106	2.2326	-1.65216	-1.6335	26.86	-22.59	-0.12 - 0.00321 - 0.00321	
4	-7.5000	.015625	712.62	616.33	1.8053	2.4651	5.4355	-11.3346	93.67	-93.67	-0.27 - 0.00343 - 0.002499	
WAVE												
0	-1.0000	.0032500	8.99	5.89	-3.6932	5.8832	-3.1939	-2.3285	46.61	-62.68	-1.69	-0.00377 - 0.00146
1	-1.0000	.0032500	238.49	120.73	-1.9210	5.7409	-1.7409	-1.8775	42.79	-55.59	-0.35 - 0.00379 - 0.00246	
2	-1.0000	.0032500	437.95	261.75	-1.8077	-1.6679	-2.1320	-1.2960	46.63	-45.94	-0.64 - 0.00363 - 0.00199	
3	-1.0000	.0032500	616.83	436.93	1.6106	-1.6397	-1.7281	-1.2781	36.87	-22.59	-0.95 - 0.00361 - 0.001975	
4	-1.0000	.0032500	712.77	616.69	-1.2556	-1.1931	-5.4790	-13.8703	93.67	-93.67	-0.62 - 0.00371 - 0.002275	
WAVE												
0	-1.2500	.0032500	8.99	5.89	-4.4629	5.8832	-4.4629	-2.7915	59.29	-63.09	-1.63	-0.00389 - 0.00166
1	-1.2500	.0032500	238.49	120.73	-1.9210	5.7409	-1.7409	-1.8775	42.79	-55.59	-0.35 - 0.00379 - 0.00246	
2	-1.2500	.0032500	437.95	261.75	-1.8077	-1.6679	-2.1320	-1.2960	46.63	-45.94	-0.64 - 0.00363 - 0.00199	
3	-1.2500	.0032500	616.83	436.93	1.6106	-1.6397	-1.7281	-1.2781	36.87	-22.59	-0.95 - 0.00361 - 0.001975	
4	-1.2500	.0032500	712.77	616.69	-1.2556	-1.1931	-5.4790	-13.8703	93.67	-93.67	-0.62 - 0.00371 - 0.002275	
WAVE												
0	-1.7500	.0032500	8.99	5.89	-5.5916	5.8832	-5.5916	-2.3285	47.34	-63.49	-1.52	-0.00393 - 0.000970
1	-1.7500	.0032500	238.49	120.73	-1.9210	5.7409	-1.7409	-1.8775	39.17	-55.57	-0.19 - 0.00377 - 0.00154	
2	-1.7500	.0032500	437.95	261.75	-1.8077	-1.6679	-2.1320	-1.2960	46.63	-45.94	-0.21 - 0.00363 - 0.001610	
3	-1.7500	.0032500	616.83	436.93	1.6106	-1.6397	-1.7281	-1.2781	36.87	-22.59	-0.16 - 0.00362 - 0.001626	
4	-1.7500	.0032500	712.77	616.69	-1.2556	-1.1931	-5.4790	-13.8703	93.67	-93.67	-0.22 - 0.00371 - 0.002275	
WAVE												
0	-1.7500	.015625	8.99	5.89	-5.5916	5.8832	-5.5916	-2.3285	47.34	-63.49	-1.52	-0.00393 - 0.000970
1	-1.7500	.015625	238.49	120.73	-1.9210	5.7409	-1.7409	-1.8775	39.17	-55.57	-0.19 - 0.00377 - 0.00154	
2	-1.7500	.015625	437.95	261.75	-1.8077	-1.6679	-2.1320	-1.2960	46.63	-45.94	-0.21 - 0.00363 - 0.001610	
3	-1.7500	.015625	616.83	436.93	1.6106	-1.6397	-1.7281	-1.2781	36.87	-22.59	-0.16 - 0.00362 - 0.001626	
4	-1.7500	.015625	712.77	616.69	-1.2556	-1.1931	-5.4790	-13.8703	93.67	-93.67	-0.22 - 0.00371 - 0.002275	
WAVE												
0	-2.0000	.0032500	8.99	5.89	-5.9517	5.8832	-5.9517	-2.3285	48.66	-64.17	-1.17	-0.00364 - 0.000977
1	-2.0000	.0032500	238.49	120.73	-1.9210	5.7409	-1.7409	-1.8775	48.33	-55.56	-0.32 - 0.00366 - 0.00146	
2	-2.0000	.0032500	437.95	261.75	-1.8077	-1.6679	-2.1320	-1.2960	46.63	-45.94	-0.97 - 0.00369 - 0.001610	
3	-2.0000	.0032500	616.83	436.93	1.6106	-1.6397	-1.7281	-1.2781	36.87	-22.59	-0.16 - 0.00362 - 0.001626	
4	-2.0000	.0032500	712.77	616.69	-1.2556	-1.1931	-5.4790	-13.8703	93.67	-93.67	-0.22 - 0.00371 - 0.002275	

TABLE 4 – (continued)

WAVE	2.2590	-0.31250	1.356	9.88	-6.2850	-0.9823	-0.6176	-3.0992	56.10	-1.4603	-0.91270	-0.01163
0	2.2590	-0.31250	1.356	9.88	-6.2850	-0.9823	-0.6176	-3.0992	56.10	-1.4603	-0.91270	-0.01163
1	2.2590	-0.31250	227.30	117.92	-2.1118	-2.5823	-2.1069	-0.9823	55.50	-0.91270	-0.01163	-0.00060
2	2.2590	-0.31250	635.37	259.36	-3.4596	-2.4596	-2.4596	-2.4596	60.30	-0.91270	-0.01163	-0.00060
3	2.2590	-0.31250	616.52	412.31	-2.5596	-3.7910	-3.7910	-3.7910	31.00	-0.91270	-0.01163	-0.00060
4	2.2590	-0.31250	713.27	692.91	-1.6329	-0.1337	-0.1337	-0.1337	0.00	-0.91270	-0.01163	-0.00060
WAVE	2.5600	-0.31250	6.0	1.880	-6.2432	-0.9880	-0.6080	-3.0970	61.99	-1.4603	-0.91581	-0.00495
0	2.5600	-0.31250	6.0	1.880	-6.2432	-0.9880	-0.6080	-3.0970	61.99	-1.4603	-0.91581	-0.00495
1	2.5600	-0.31250	226.25	117.20	-4.8926	-2.6859	-2.2492	-0.9880	60.97	-0.91581	-0.00495	-0.00160
2	2.5600	-0.31250	256.50	116.69	-3.2238	-0.1337	-0.1337	-0.1337	60.95	-0.91581	-0.00495	-0.00160
3	2.5600	-0.31250	613.13	311.27	-2.1997	-0.4992	-0.4992	-0.4992	30.79	-0.91581	-0.00495	-0.00160
4	2.5600	-0.31250	710.31	661.80	-1.8898	-0.7970	-0.7970	-0.7970	0.00	-0.91581	-0.00495	-0.00160
WAVE	2.7500	-0.31250	-1.56	9.88	-6.2550	-0.9823	-0.6176	-3.0992	65.32	-1.4603	-0.91680	-0.00391
0	2.7500	-0.31250	-1.56	9.88	-6.2550	-0.9823	-0.6176	-3.0992	65.32	-1.4603	-0.91680	-0.00391
1	2.7500	-0.31250	226.10	116.69	-5.3487	-2.5902	-2.7084	-0.1360	52.03	-0.91680	-0.00391	-0.00160
2	2.7500	-0.31250	611.09	277.71	-3.6275	-1.4294	-1.3160	-0.3160	35.00	-0.91680	-0.00391	-0.00160
3	2.7500	-0.31250	613.04	416.00	-3.1904	-0.9604	-0.9604	-0.9604	27.00	-0.91680	-0.00391	-0.00160
4	2.7500	-0.31250	714.31	660.52	-2.4223	-0.4922	-0.4922	-0.4922	0.00	-0.91680	-0.00391	-0.00160
WAVE	3.0080	-0.31250	-3.09	9.51	-5.9757	-1.9616	-1.2200	-3.7956	66.37	-1.4603	-0.91720	-0.00056
0	3.0080	-0.31250	-3.09	9.51	-5.9757	-1.9616	-1.2200	-3.7956	66.37	-1.4603	-0.91720	-0.00056
1	3.0080	-0.31250	225.30	115.90	-5.2077	-2.4792	-2.9657	-0.5916	52.05	-0.91720	-0.00056	-0.00033
2	3.0080	-0.31250	418.72	256.77	-4.7937	-0.8737	-0.8737	-0.8737	46.93	-0.91720	-0.00056	-0.00033
3	3.0080	-0.31250	709.76	659.12	-3.2374	-0.3958	-0.3958	-0.3958	36.52	-0.91720	-0.00056	-0.00033
4	3.0080	-0.31250	710.76	659.12	-2.2597	-0.7626	-0.7626	-0.7626	0.00	-0.91720	-0.00056	-0.00033
WAVE	3.7500	-0.31250	-3.09	9.51	-5.9808	-2.0525	-1.7923	-3.5176	66.25	-1.4603	-0.91759	-0.00165
0	3.7500	-0.31250	-3.09	9.51	-5.9808	-2.0525	-1.7923	-3.5176	66.25	-1.4603	-0.91759	-0.00165
1	3.7500	-0.31250	222.00	115.37	-5.1741	-2.4794	-2.8794	-0.6054	61.70	-0.91759	-0.00165	-0.00048
2	3.7500	-0.31250	612.00	422.00	-4.2746	-0.9679	-0.9679	-0.9679	56.97	-0.91759	-0.00165	-0.00048
3	3.7500	-0.31250	613.04	423.00	-3.6237	-0.4922	-0.4922	-0.4922	42.61	-0.91759	-0.00165	-0.00048
4	3.7500	-0.31250	710.00	651.39	-2.3580	-0.9612	-0.9612	-0.9612	0.00	-0.91759	-0.00165	-0.00048
WAVE	3.7500	-0.31250	-5.16	8.9	-5.9808	-2.0525	-1.7923	-3.5176	66.25	-1.4603	-0.91759	-0.00165
0	3.7500	-0.31250	-5.16	8.9	-5.9808	-2.0525	-1.7923	-3.5176	66.25	-1.4603	-0.91759	-0.00165
1	3.7500	-0.31250	222.00	115.37	-5.1741	-2.4794	-2.8794	-0.6054	61.70	-0.91759	-0.00165	-0.00048
2	3.7500	-0.31250	612.00	422.00	-4.2746	-0.9679	-0.9679	-0.9679	56.97	-0.91759	-0.00165	-0.00048
3	3.7500	-0.31250	613.04	423.00	-3.6237	-0.4922	-0.4922	-0.4922	42.61	-0.91759	-0.00165	-0.00048
4	3.7500	-0.31250	709.19	651.39	-2.3580	-0.9612	-0.9612	-0.9612	0.00	-0.91759	-0.00165	-0.00048
WAVE	3.7500	-0.31250	-5.16	8.9	-5.9808	-2.0525	-1.7923	-3.5176	66.25	-1.4603	-0.91759	-0.00165
0	3.7500	-0.31250	-5.16	8.9	-5.9808	-2.0525	-1.7923	-3.5176	66.25	-1.4603	-0.91759	-0.00165
1	3.7500	-0.31250	222.00	115.37	-5.1741	-2.4794	-2.8794	-0.6054	61.70	-0.91759	-0.00165	-0.00048
2	3.7500	-0.31250	612.00	422.00	-4.2746	-0.9679	-0.9679	-0.9679	56.97	-0.91759	-0.00165	-0.00048
3	3.7500	-0.31250	613.04	423.00	-3.6237	-0.4922	-0.4922	-0.4922	42.61	-0.91759	-0.00165	-0.00048
4	3.7500	-0.31250	709.19	651.39	-2.3580	-0.9612	-0.9612	-0.9612	0.00	-0.91759	-0.00165	-0.00048
WAVE	3.7500	-0.31250	-7.07	5.07	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
0	3.7500	-0.31250	-7.07	5.07	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
1	3.7500	-0.31250	219.17	114.35	-5.0800	-1.7850	-0.6225	-1.2227	58.01	-0.91860	-0.00060	-0.00037
2	3.7500	-0.31250	625.03	253.00	-4.6377	-3.3505	-5.2780	-0.6226	52.22	-0.91860	-0.00060	-0.00037
3	3.7500	-0.31250	629.07	424.77	-4.2746	-2.5591	-5.7931	-0.7611	46.96	-0.91860	-0.00060	-0.00037
4	3.7500	-0.31250	708.93	654.72	-2.3580	-0.2391	-0.3919	-0.2391	32.36	-0.91860	-0.00060	-0.00037
WAVE	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
0	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
1	4.0000	-0.31250	219.17	114.35	-5.0800	-1.7850	-0.6225	-1.2227	58.01	-0.91860	-0.00060	-0.00037
2	4.0000	-0.31250	625.03	253.00	-4.6377	-3.3505	-5.2780	-0.6226	52.22	-0.91860	-0.00060	-0.00037
3	4.0000	-0.31250	629.07	424.77	-4.2746	-2.5591	-5.7931	-0.7611	46.96	-0.91860	-0.00060	-0.00037
4	4.0000	-0.31250	708.93	654.72	-2.3580	-0.2391	-0.3919	-0.2391	32.36	-0.91860	-0.00060	-0.00037
WAVE	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
0	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
1	4.0000	-0.31250	219.17	114.35	-5.0800	-1.7850	-0.6225	-1.2227	58.01	-0.91860	-0.00060	-0.00037
2	4.0000	-0.31250	625.03	253.00	-4.6377	-3.3505	-5.2780	-0.6226	52.22	-0.91860	-0.00060	-0.00037
3	4.0000	-0.31250	629.07	424.77	-4.2746	-2.5591	-5.7931	-0.7611	46.96	-0.91860	-0.00060	-0.00037
4	4.0000	-0.31250	708.93	654.72	-2.3580	-0.2391	-0.3919	-0.2391	32.36	-0.91860	-0.00060	-0.00037
WAVE	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
0	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
1	4.0000	-0.31250	219.17	114.35	-5.0800	-1.7850	-0.6225	-1.2227	58.01	-0.91860	-0.00060	-0.00037
2	4.0000	-0.31250	625.03	253.00	-4.6377	-3.3505	-5.2780	-0.6226	52.22	-0.91860	-0.00060	-0.00037
3	4.0000	-0.31250	629.07	424.77	-4.2746	-2.5591	-5.7931	-0.7611	46.96	-0.91860	-0.00060	-0.00037
4	4.0000	-0.31250	708.93	654.72	-2.3580	-0.2391	-0.3919	-0.2391	32.36	-0.91860	-0.00060	-0.00037
WAVE	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
0	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
1	4.0000	-0.31250	219.17	114.35	-5.0800	-1.7850	-0.6225	-1.2227	58.01	-0.91860	-0.00060	-0.00037
2	4.0000	-0.31250	625.03	253.00	-4.6377	-3.3505	-5.2780	-0.6226	52.22	-0.91860	-0.00060	-0.00037
3	4.0000	-0.31250	629.07	424.77	-4.2746	-2.5591	-5.7931	-0.7611	46.96	-0.91860	-0.00060	-0.00037
4	4.0000	-0.31250	708.93	654.72	-2.3580	-0.2391	-0.3919	-0.2391	32.36	-0.91860	-0.00060	-0.00037
WAVE	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
0	4.0000	-0.31250	-6.91	5.08	-6.4629	-0.6429	-2.7915	-2.7915	61.00	-1.4603	-0.91860	-0.00060
1	4.0000	-0.31250	219.17	114.35	-5.0800	-1.7850	-0.6225	-1.2227	58.01	-0.91860	-0.00060	-0.00037
2	4.0000	-0.31250	625.03	253.00	-4.6377	-3.3505	-5.2780	-0.6226	52.22	-0.91860	-0.00060	-0.00037
3	4.0000											

TABLE 4 - (continued)

WAVE	36.0000	-0.15625	-0.09	-5.86	3.4932	-3.0032	3.1639	2.3205
0	36.0000	-0.15625	-0.09	-5.86	3.4932	-3.0032	3.1639	2.3205
1	36.0000	-0.15625	23.25	1.1572	9.986	1.2055	37.952	1.0006
2	36.0000	-0.15625	267.59	6.992	2.064	3.3868	35.385	49.17
3	36.0000	-0.15625	612.64	4.297	6.631	2.6273	36.958	46.65
4	36.0000	-0.15625	642.34	1.1622	5.536	1.6389	6.1932	91.09
WAVE	37.0000	-0.15625	-3.09	-7.91	5.9757	-1.9616	1.2289	3.7946
0	37.0000	-0.15625	-2.99	-7.51	5.9757	-1.9616	1.2289	3.7946
1	37.0000	-0.15625	23.04	1.1757	3.2276	2.3499	20.396	1.7346
2	37.0000	-0.15625	622.62	2.6679	3.2612	2.2488	2.2367	1.6428
3	37.0000	-0.15625	612.84	2.5065	2.9432	2.0795	1.7672	13.73
4	37.0000	-0.15625	644.49	1.3497	2.3465	0.4776	2.1791	.09
W, VE	38.0000	-0.15625	1.89	-7.51	5.9757	-1.9616	-1.2199	3.7946
0	38.0000	-0.15625	1.89	-7.51	5.9757	-1.9616	-1.2199	3.7946
1	38.0000	-0.15625	21.59	1.2814	5.4898	2.6795	1.2685	1.7346
2	38.0000	-0.15625	42.87	25.66	4.9536	3.4930	7.965	1.6428
3	38.0000	-0.15625	612.79	616.57	6.1759	4.3320	7.216	10.19
4	38.0000	-0.15625	786.74	646.84	2.1793	5.1864	.3917	.9175
WAVE	39.0000	-0.15625	0.99	-5.86	3.4932	5.0032	-3.1939	2.3205
0	39.0000	-0.15625	0.99	-5.86	3.4932	5.0032	-3.1939	2.3205
1	39.0000	-0.15625	22.37	1.2258	5.4898	2.6795	1.2685	1.7346
2	39.0000	-0.15625	755.47	643.02	3.5598	1.1858	5.3377	10.19
3	39.0000	-0.15625	621.94	616.84	4.4472	4.9353	6.925	11.15
4	39.0000	-0.15625	656.17	2.2380	5.2593	0.3767	0.0332	.09
WAVE	40.0000	-0.15625	10.80	-0.98	-0.98	6.2882	-3.0497	.0008
0	40.0000	-0.15625	10.80	-0.98	-0.98	6.2882	-3.0497	.0008
1	40.0000	-0.15625	22.18	1.2258	3.761	1.3759	-1.1215	-1.1196
2	40.0000	-0.15625	437.79	437.79	2.5662	2.1936	-2.0963	-2.0961
3	40.0000	-0.15625	622.38	624.89	2.3662	1.5999	-2.6379	-2.6328
4	40.0000	-0.15625	712.93	750.62	3.2793	-1.2793	-1.2822	-1.2822
WAVE	41.0000	-0.15625	0.09	5.86	-3.4932	5.0032	-3.1939	2.3205
0	41.0000	-0.15625	0.09	5.86	-3.4932	5.0032	-3.1939	2.3205
1	41.0000	-0.15625	22.18	1.2258	3.761	1.3759	-1.1215	-1.1196
2	41.0000	-0.15625	437.79	437.79	2.5662	2.1936	-2.0963	-2.0961
3	41.0000	-0.15625	622.38	624.89	2.3662	1.5999	-2.6379	-2.6328
4	41.0000	-0.15625	712.93	750.62	3.2793	-1.2793	-1.2822	-1.2822
WAVE	42.0000	-0.15625	0.09	5.86	-3.4932	5.0032	-3.1939	2.3205
0	42.0000	-0.15625	0.09	5.86	-3.4932	5.0032	-3.1939	2.3205
1	42.0000	-0.15625	22.18	1.2258	3.761	1.3759	-1.1215	-1.1196
2	42.0000	-0.15625	437.79	437.79	2.5662	2.1936	-2.0963	-2.0961
3	42.0000	-0.15625	622.38	624.89	2.3662	1.5999	-2.6379	-2.6328
4	42.0000	-0.15625	712.93	750.62	3.2793	-1.2793	-1.2822	-1.2822
WAVE	43.0000	-0.15625	0.09	5.86	-3.4932	5.0032	-3.1939	2.3205
0	43.0000	-0.15625	0.09	5.86	-3.4932	5.0032	-3.1939	2.3205
1	43.0000	-0.15625	22.18	1.2258	3.761	1.3759	-1.1215	-1.1196
2	43.0000	-0.15625	437.79	437.79	2.5662	2.1936	-2.0963	-2.0961
3	43.0000	-0.15625	622.38	624.89	2.3662	1.5999	-2.6379	-2.6328
4	43.0000	-0.15625	712.93	750.62	3.2793	-1.2793	-1.2822	-1.2822
WAVE	44.0000	-0.15625	0.09	5.86	-3.4932	5.0032	-3.1939	2.3205
0	44.0000	-0.15625	0.09	5.86	-3.4932	5.0032	-3.1939	2.3205
1	44.0000	-0.15625	22.18	1.2258	3.761	1.3759	-1.1215	-1.1196
2	44.0000	-0.15625	437.79	437.79	2.5662	2.1936	-2.0963	-2.0961
3	44.0000	-0.15625	622.38	624.89	2.3662	1.5999	-2.6379	-2.6328
4	44.0000	-0.15625	712.93	750.62	3.2793	-1.2793	-1.2822	-1.2822

TABLE 4 - (continued)

Figure 5 shows the tension in the upper cable segment for $0 \leq t \leq 20$ for Cases 1A to 1E, where C_1 increases from 2.4×10^1 lb (1.07×10^2 N) to 2.4×10^5 lb (1.07×10^6 N). As would be expected, the figure shows that tension fluctuations increase with increasing values of C_1 . At the highest value of $C_1 = 2.4 \times 10^5$ lb (1.07×10^6 N), Figure 5 shows that the tension is zero at $t = 0.25$ sec (i.e., the cable in slack) and then jumps to a value of 89.47 lb (398.1 N) at $t = 0.5$ sec. This sudden jump in tension after a slack condition is often referred to as "snap loading." At the other extreme, the tension shows a peak-to-trough fluctuation of less than 2.5 lb (11.1 N) at the lowest value of $C_1 = 2.4 \times 10^1$ lb (1.07×10^2 N).

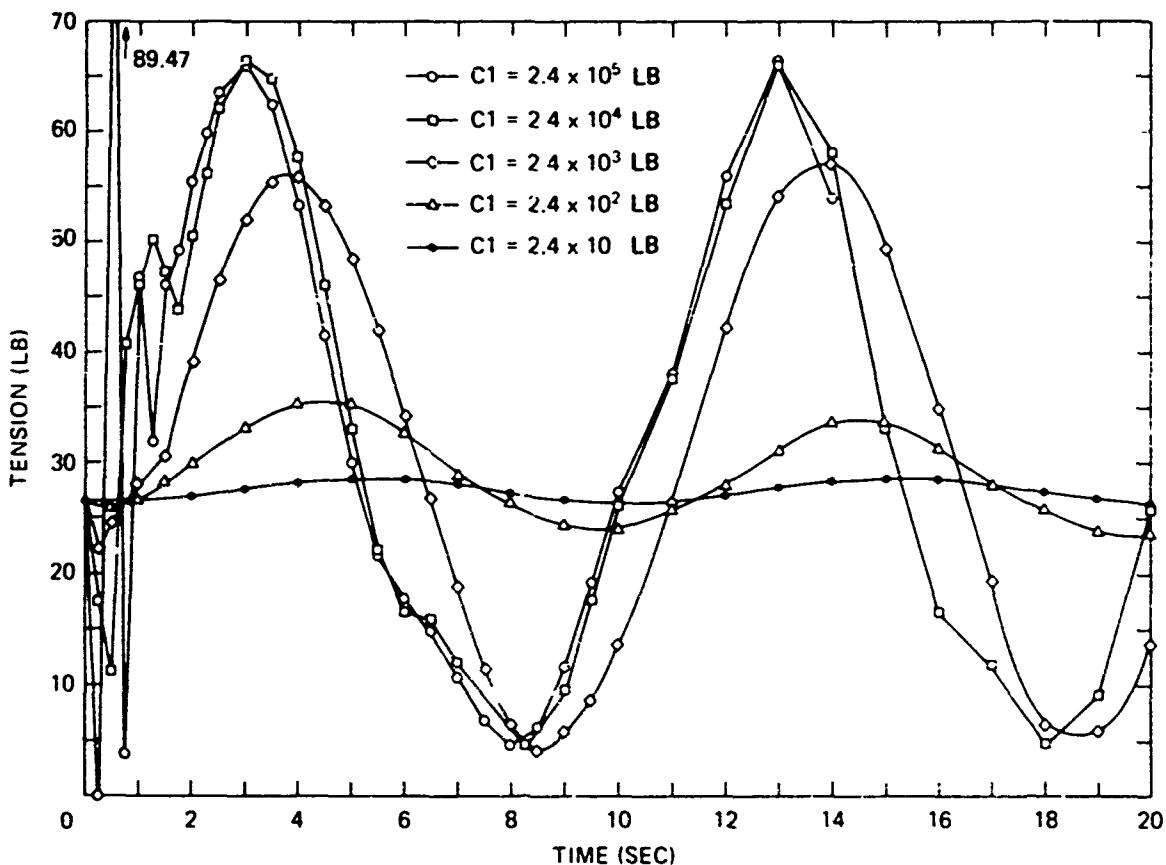


Figure 5 – Tension in Upper Cable Segment
for Various Values of C_1

Table 2 indicates that the computer execution time remains at approximately 60 sec for Cases 1A, 1B, and 1C for which $C_1 \leq 2.4 \times 10^3$ lb (1.07×10^4 N). At higher values of C_1 , Cases 1D and 1E, execution time increases with increasing C_1 . Execution time also increases with decreasing values of C_2 , Case 1F. The reasons for these increases may be most conveniently explained by considering Equation (9) which shows that for a given value of

$(T - T_0)$. ϵ decreases with increasing values of C1 and decreasing values of C2. Small values of ϵ , in turn, mean that the x- and y-displacements must be calculated with greater precision, leading to smaller integration time steps.

PROBLEM 2 – UNIFORM CABLE REPRESENTED BY VARIOUS NUMBERS OF SEGMENTS

Problem: Solve Problem 1D with the single exception of representing the cable by the following number of nodes, NCAB-1: 1, 2, 4, 8, and 15. Consider the nodes to be equally spaced.

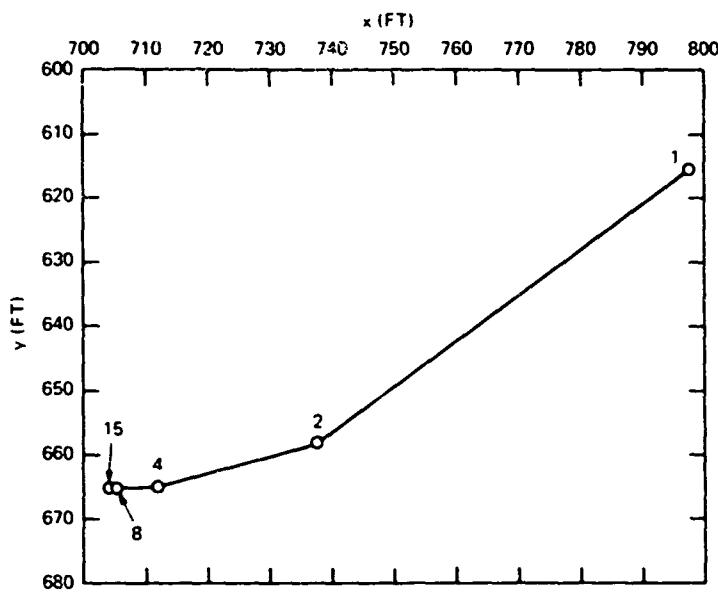
Solution: The data cards for this problem are listed in Table 5. Cards 1 and 4 to 12, which are identical to those for Problem 1D, are omitted. Also omitted are Cards 14 to 33 which are similar to those for Problem 1D with the exception that the number of entries depends on NCAB.

TABLE 5 – INPUT DATA FOR SAMPLE PROBLEM 2

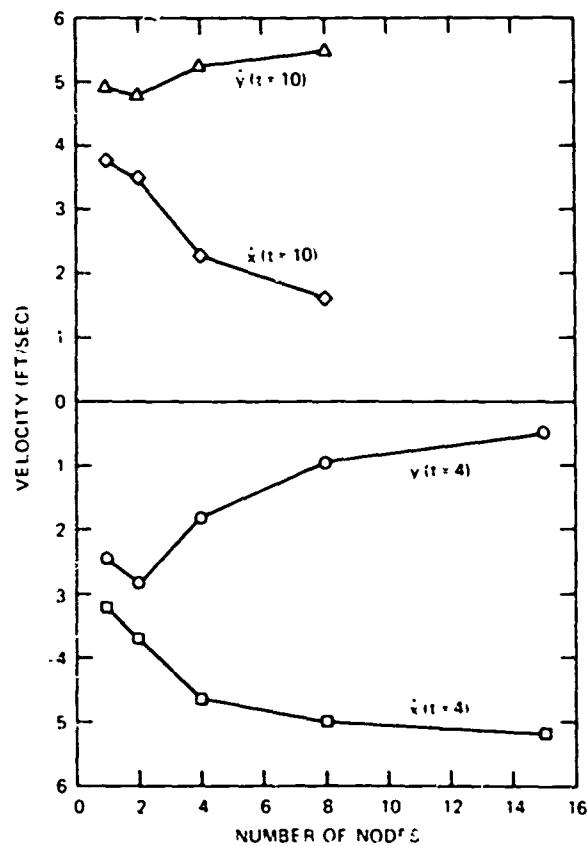
	1	11	21	31	41	51	61	71
Card 2	bbb PROBLEM 2A, NCAB-1 = 1							
Card 3	bb1bb1bb2bb2							
Card 13	1000.							
Card 2	bbb PROBLEM 2B, NCAB-1 = 2							
Card 3	bb1bb1bb3bb2							
Card 13	500. 500.							
Card 2	bbb PROBLEM 2C, NCAB-1 = 4							
Card 3	bb1bb1bb5bb2							
Card 13	250. 250. 250. 250.							
Card 2	bbb PROBLEM 2D, NCAB-1 = 8							
Card 3	bb1bb1bb9bb2							
Card 13	125. 125. 125. 125. 125. 125. 125. 125.							
Card 13	bbb							
Card 2	bbb PROBLEM 2E, NCAB-1 = 15							
Card 3	bb1bb1b16bb2							
Card 13	66.667 66.667 66.667 66.667 66.667 66.667 66.667 66.667							
Card 13	66.667 66.667 66.667 66.667 66.667 66.667 66.667 66.667							

Figure 6 shows how the steady-state location of the lower weight varies with NCAB-1. The results are poor for $NCAB-1 \leq 2$, good for $NCAB-1 = 4$, and converged for $NCAB-1 \geq 8$.

Figure 7 shows the velocities \dot{x} and \dot{y} of the lower unit at $t = 4$ and 10 for various values of NCAB-1. The results suggest that a minimum of four nodes is required to obtain maximum



**Figure 6 – Steady-State Location of Lower Weight
for Various Numbers of Nodes**



**Figure 7 – Velocity of Lower Weight for
Various Numbers of Nodes**

velocities with accuracies to within 10-20 percent. The magnitudes of the computed velocities generally increase with increasing number of nodes.

Table 2 shows the manner in which the computer execution time ET increases with the number of nodes. The execution times are approximately proportional to NCAB squared

$$ET \propto (NCAB)^2 \quad (56)$$

PROBLEM 3 – DEPLOYMENT OF SUSPENDED CABLE IN CIRCULATING WATER CHANNEL

Problem: Compute the deployment of an initially vertical cable, suspended in the DTNSRDC Circulating Water Channel, to its final steady-state configuration in the presence of channel flow speeds of 1, 2, and 4 knots (0.515, 1.03, 2.06 m/s). The fixed cable and lower body parameters are as follows:

Cable:

length	12 ft (3.66 m)
diameter	0.12 in. (0.305 cm)
normal drag coefficient	1.4
tangential drag coefficient	0.02
weight in fluid	0
mass	0.0002 slugs/ft (0.00957 kg/m)
reference tension	2 lb (8.9 N)
C_1	20 lb (89. N)
C_2	1.
internal damping coefficient	0.

Lower weight:

weight in fluid	2 lb (8.9 N)
drag area in x-direction	0.05 ft ² (0.00465 m ²)
drag area in y-direction	0.05 ft ² (0.00465 m ²)
virtual mass in x-direction	0.1 slugs (1.46 kg)
virtual mass in y-direction	0.1 slugs (1.46 kg)

Fluid density:

1.94 slugs/ft³ (1000.6 kg/m³)

Represent the cable by four equal segments, not including the fictitious cable segment below the lower weight. Take the total time interval to be 20 sec. For the initial interval of 1 sec, printout of the transient cable configuration is desired every 0.1 sec. For the final 19 sec, increase the printout interval to 0.2 sec. Let the system start from rest with the initial tension in each cable segment equal to 2 lb (8.9 N).

Solution: The data cards for this problem are listed in Table 6. The cards which are the same for all the cases are listed at the top of the table. The cards for the title and the current magnitude which differ for each case are listed at the bottom of the table.

TABLE 6 – INPUT DATA FOR SAMPLE PROBLEM 3

	1	11	21	31	41	51	61	71
Card 1	bb1							
Card 3	bb1bb1bb5bb2							
Card 4	0.1							
Card 5	0.							
Card 6	0.							
Card 7	0.							
Card 8	0.							
Card 9	0.1							
Card 10	0.							
Card 11	1.94	0.	0.	-15.	0.	1.0	1.0	0.
Card 12	1.	0.1	20.	0.2	-1.	0.	99999.	
Card 13	3.	3.	3.	3.				
Card 14	0.12	0.12	0.12	0.12				
Card 15	1.4	1.4	1.4	1.4				
Card 16	0.02	0.02	0.02	0.02				
Card 17	0.	0.	0.	0.				
Card 18	0.0002	0.0002	0.0002	0.0002				
Card 19	2.	2.	2.	2.				
Card 20	20.	20.	20.	20.				
Card 21	1.0	1.0	1.0	1.0				
Card 22	0.	0.	0.	0.				
Card 23	0.	0.	0.	2.				
Card 24	0.	0.	0.	0.05				
Card 25	0.	0.	0.	0.05				
Card 26	0.	0.	0.	0.1				
Card 27	0.	0	0.	0.1				
Card 28	0.	100.						
Card 30	0.	0.	0.	0.				
Card 31	2.	2.	2.	2.				
Card 32	0.	0.	0.	0.				
Card 33	0.	0.	0.	0.				

TABLE 6 - (continued)

	1	11	21	31	41	51	61	71
Card 2	bbb PROBLEM 3A, V = 1 KNOT							
Card 29	1.0	1.0						
Card 2	bbb PROBLEM 3B, V = 2 KNOTS							
Card 29	2.0	2.0						
Card 2	bbb PROBLEM 3C, V = 4 KNOTS							
Card 29	4.0	4.0						

Figures 8, 9, and 10 show the configuration of the cable nodes at various times during deployment for currents of 1, 2, and 4 knots (0.515, 1.03, 2.06 m/s), respectively. These figures show that approximately 8 sec are required to reach the steady-state configuration.

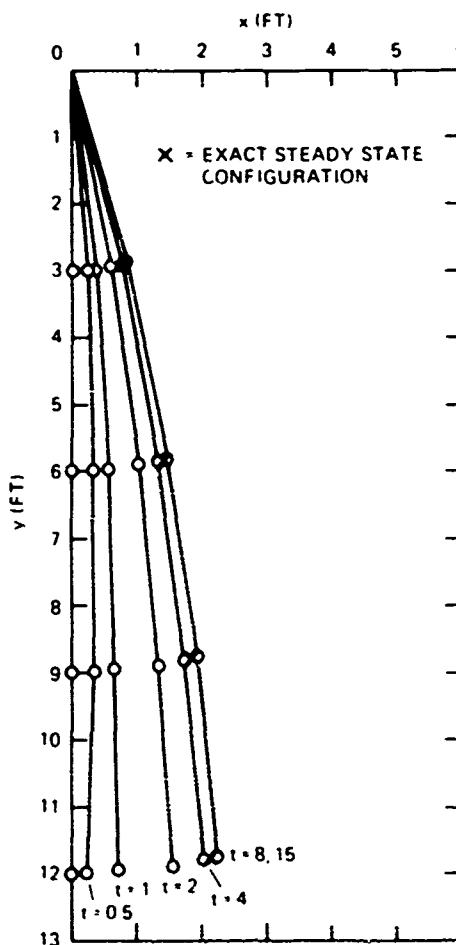


Figure 8 - Configuration of Nodes at Various Times during Deployment.
C = 1 Knot

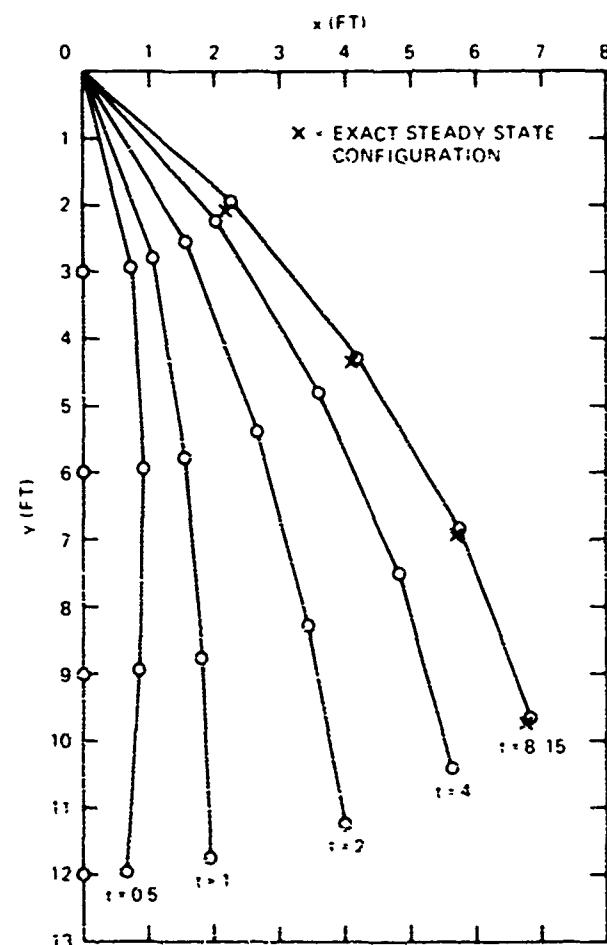
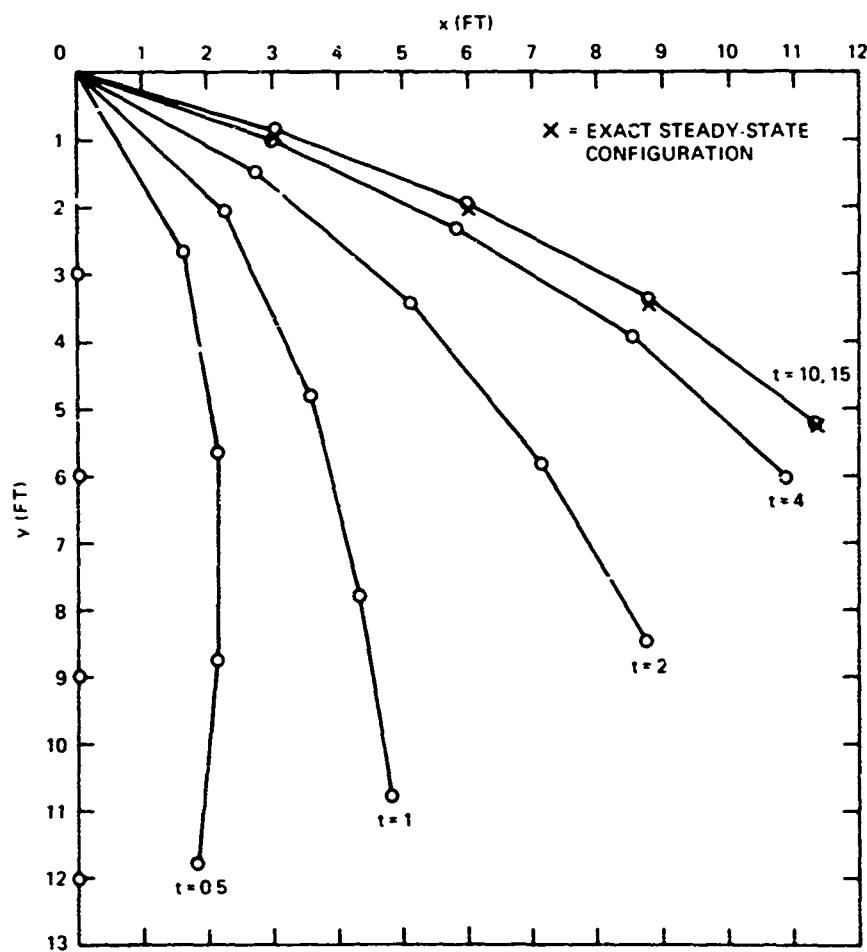


Figure 9 - Configuration of Nodes at Various Times during Deployment.
C = 2 Knots



**Figure 10 – Configuration of Nodes at Various Times
during Deployment, C = 4 Knots**

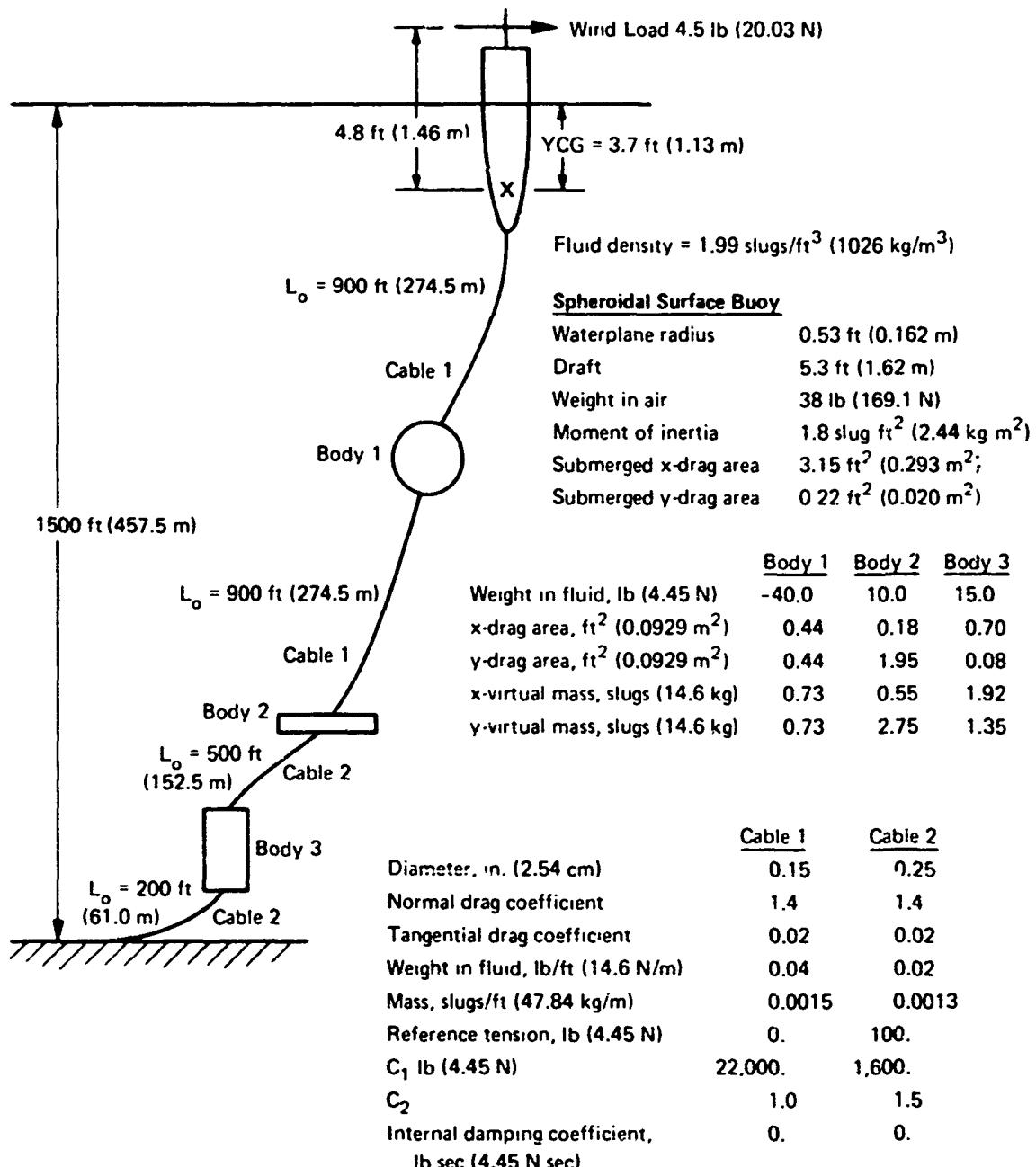
In all three cases, the final deployed configuration modeled by the four nodes was in close agreement with the exact configuration calculated by using the differential equations (1) to (5). It is of interest to note that because of its large inertia, the lower weight characteristically lags during the initial instants of deployment.

Table 2 shows that the computer execution time for a total of 20 sec of deployment time, which is well in excess of the 8 sec required to reach the steady-state configuration, is approximately 100 sec.

PROBLEM 4 – COMPLETE MOORED BUOY-CABLE-BODY SYSTEM

Problem: Compute the dynamic motions for the moored buoy-cable-body system, with parameters as shown in Table 7.

TABLE 7 – PARAMETERS FOR MOORED BUOY-CABLE-BODY SYSTEM



Current Profile:	Depth (ft) (0.305 m)	Current (knots) (0.515 m/s)
	0.	2.50
	500.	1.30
	1000.	0.50
	1500.	0.50

The upper body is a buoyancy bag, the middle body is a damping disk, and the lower body is an acoustic unit. Consider the following three cases:

Case A. Use the formulation contained in the program for a spheroidal buoy. Let the surface wave be composed of a single component with frequency = 0.1 cps, amplitude = 7.5 ft (2.29 m), and phase = 0 degrees.

Case B. Same as Case A except that the buoy is read in as a spar buoy, modeled by 11 cross-sectional areas as a function of depth from 0 to 5.3 ft (1.62 m).

Case C. Same as Case A except that the surface wave amplitudes are to be calculated by using the Pierson-Moskowitz sea spectrum for six components. Take the significant wave height as 15 ft (4.58 m), the range of frequencies from 0.04 to 0.28 cps, and the phase of the lowest frequency component equal to -60 deg.

For all three cases, model the cable by 4 nodes, with three of them corresponding to the three intermediate bodies and the extra node 200 ft of cable below the surface buoy. Compute the dynamic motions for 50 sec. For the initial 10 sec, print out the dynamic motions every 0.1 sec. For the final 40 sec, increase the printout interval to 0.5 sec. Let the system start from rest with the initial angle and tension of each segment equal to the steady-state values. Let the initial x-displacement of the buoy center of gravity be 7.5 ft (2.29 m).

Solution: The data cards for this problem are listed in Table 8. Again, cards which are the same for all the cases are listed at the top of the table. The cards for the title surface buoy, and surface waves, which differ for each case, are listed at the bottom of the table.

Figure 11 shows the pitch angle ψ of the surface buoy for all three cases for $20 \leq t \leq 40$. Perhaps the principal feature is the comparison of the results for the same surface buoy treated as a spheroidal buoy and as a spar buoy. The figure shows that when the spheroidal buoy formulation was used, pitch results were about 1 to 2 deg higher than those obtained from the spar buoy formulation. This discrepancy is due to the difference in the added inertia terms for the two formulations (compare Equations (27), (28), (32) and Table 1).

The figure also shows that results for the single frequency cases 4A and 4B exhibit a periodic behavior with a period of 10 sec; the results for the multifrequency case 4C show a more random behavior.

Table 2 shows that the execution times for the single-frequency cases 4A and 4B were approximately 80 sec. This time was increased to 100 sec for Case 4C, where the program must calculate six components to obtain the surface wave.

TABLE 8 – INPUT DATA FOR SAMPLE PROBLEM 4

	1	11	21	31	41	51	61	71
Card 1	bb1							
Card 11	1.99	0.	4.5	-100.	3.15	1.0	1.0	0
Card 12	10.	0.1	50.	0.5	1.0	0.	99999.	
Card 13	200.	700.	900.	500.	200.			
Card 14	0.15	0.15	0.15	0.25	0.25			
Card 15	1.4	1.4	1.4	1.4	1.4			
Card 16	0.02	0.02	0.02	0.02	0.02			
Card 17	0.04	0.04	0.04	0.02	0.02			
Card 18	0.0015	0.0015	0.0015	0.0013	0.0013			
Card 19	0.	0.	0.	100.	100.			
Card 20	22000.	22000.	22000.	1600.	1600.			
Card 21	1.0	1.0	1.0	1.5	1.5			
Card 22	0.	0.	0.	0.	0.			
Card 23	0.	-40.	10.	15.	0.			
Card 24	0.	0.44	0.18	0.70	0.			
Card 25	0.	0.44	1.95	0.08	0.			
Card 26	0.	0.73	0.55	1.92	0.			
Card 27	0.	0.73	2.75	1.35	0.			
Card 28	0.	500.	1000.	1500.				
Card 29	2.50	1.30	0.50	0.50				
Card 30	9999.	0.	0.	0.	0.			
Card 31	0.	0.	0.	0.	0.			
Card 32	0.	0.	0.	0.	0.			
Card 33	0.	0.	0.	0.	0.			
Card 34	0.22	38.	-4.8	0.	1.6	3.7	1.8	
Card 35	7.5	0.	999.	0.	0.	0.		
Card 2	bbb PROBLEM 4A, SPHEROIDAL BUOY, SINGLE WAVE FREQUENCY							
Card 3	bb1bb1bb5bb4bb2							
Card 4	3500.							
Card 5	0.53							
Card 6	5.3							
Card 7	0.							
Card 8	7.5							
Card 9	0.1							
Card 10	0.							

TABLE 8 - (continued)

	1	11	21	31	41	51	:	71
Card 2	bbb PROBLEM 4B, SPAR BUOY, SINGLE WAVE FREQUENCY							
Card 3	b11bb1bb5bb4bb2							
Card 4	2700.							
Card 4								
Card 5	0.883	0.852	0.812	0.757	0.687	0.600	0.498	0.380
Card 5	0.247	0.097	0.					
Card 6	0.	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Card 6	4.5	5.0	5.3					
Card 7	0.							
Card 7	0.							
Card 8	7.5							
Card 9	0.1							
Card 10	0.							
Card 2	bbb PROBLEM 4C, SPHEROIDAL BUOY, SIX WAVE FREQUENCIES							
Card 3	bb1bb6bb5bb4bb2							
Card 4	3500.							
Card 5	0.53							
Card 6	5.3							
Card 7	0.							
Card 8	1015.							
Card 9	0.04	0.28						
Card 10	-60.							

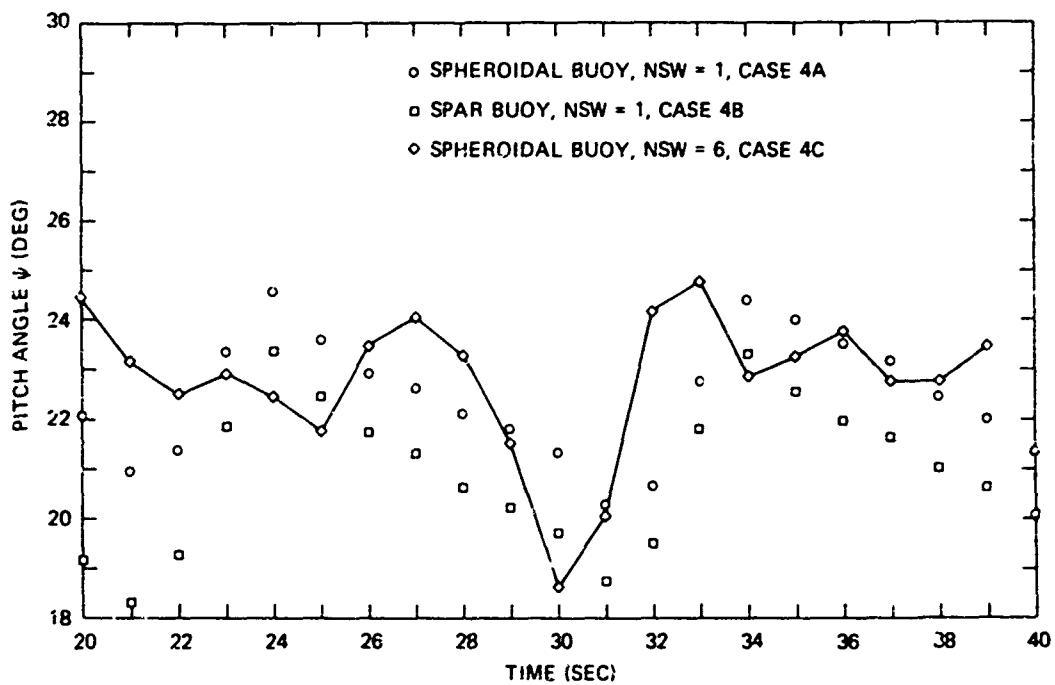


Figure 11 – Pitch of Surface Buoy

ACKNOWLEDGMENT

The author thanks Dr. K.J. Bai who performed the calculations for the added inertia coefficients of the spheroidal buoys.

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APPENDIX
LISTING OF COMPUTER PROGRAM

PROGRAM CABUDY 7674 OPT=0 ROUND=0/ TRACE FTM 4.6+420 04/19/77 11:49:22 PAGE 2
 READ(5,5) (CMI(K)) *K+1,NCAB)
 READ(5,5) (REF((K)) *K+1,NCAB)
 HEAD(5,5) (L1(K)) *K+1,NCAB)
 READ(5,2) (G2(K)) *K+1,NCAB)
 READ(5,2) (CINT((K)) *K+1,NCAB)
 READ(5,2) (WAD(K)) *K+1,NCAB)
 READ(5,2) (WADX(K)) *K+1,NCAB)
 READ(5,2) (CDABY((K)) *K+1,NCAB)
 READ(5,2) (AMB((K)) *K+1,NCAB)
 HEAD(5,2) (THBV((K)) *K+1,NCAB)
 READ(5,3) (YY((I)) *I+1,NCUR)
 READ(5,3) (PHOUL(I)) *I+1,NCAB)
 READ(5,3) (TEM(I)) *I+1,NCAB)
 HEAD(5,2) (XP1((I)) *I+1,NCAB)
 HEAD(5,2) (XP1((I)) *I+1,NCAB)
 D1WP=1AM
 JO 1000 MC=1,NCASES
 OR=U1KP
 READ(5,3) (WCA((I)) *I+1,NCUR)
 IF(MRC.LE.0) GO TO 221
 C CONVENT METRIC INPUT TO ENGLISH UNITS
 UO 222 I=1,NSH
 AXSM(I)=3.281*AXSM(I)
 IF(FSM(I)=J.28.088*AMD,FSM(I),LT.3088.) AXSM(I)=3.281*AXSM(I)
 222 AXSM(I)=J.28.0*ATSM(I)
 IF(ATSM(I)=J.E.1088.) GO TO 224
 AXSM(I)=1088.*J.281*(ASW(I))-1900.)
 224 GO TO 226
 224 COMINUE
 JO 225 I=1,NSW
 225 ASW(I)=J.281*ASW(I)
 226 RHO=0.0013388*MMO
 SUBMA=.281*SUBMATW=.281*IWX/.45281IY=.11V/.452
 COAS=J.281*I.281*QUASKTMIN=TMIN/.4.45281TB=.10H/.4.452
 TBVK=TBVK/.4.452
 JO 227 KAI,NCAB
 FLG(I)=J.281*FLG(I)*ACI(I)=0.3937*DC1(WISNC(K))=0.66846*MC(K)
 CMK(J)=.28088*CM(K)*TREF(K)/REF((K))/.45281(K)=C1(K)/.4.452
 CINT(K)=C1M(K)/.4.45281(K)=WD(K)/.4.452
 COABX(K)=J.281*3.281*COABX((K))ACABY((K)=J.281*3.281*COABY((K)
 THV((K))=0.06467*AMB((K))THBV((K))=0.06467*THBV((K))
 TEM(I)=TEM((K))/.4.45281(K)=J.281*X(I)
 227 YP1((K))=J.281*YP1((K))
 JO 228 I=1,NCUR
 YY(I)=J.281*YY(I)
 228 CLK(I)=J.281*CCK(I)/1.6874
 221 COMINUE
 1 IF(C1(NCAE).GT.0.0001) GO TO 115
 1 IF(C1(NCAE).LT.-0.0001) NCAE=0.0
 105 FLG((NCAE))=Z.*FLG((NCAE)-11WU((NCAE))=0. SCOM(NCAE)=0.
 WD1(NCAE)=0.5NC((NCAE))=0.0
 G2(INLAB)=0.5NC((NCAE))=0.0
 G2(INLAB)=1.5INT(NCAE)=0.
 115 COMINUE
 FDISM=10SM(I)
 FSM(I)=SM(I)
 PI=J.141592656*32.2
 XS1=0.
 UKUL=2.2/1.12

PROGRAM CABUOV 7/7/7 OPT=0 ROUNDOFF TKE~~0~~
 FTN 4.6+420 04/19/77 11:49:22 PAGE 3

 115 ULLL=1/3.0
 JO 151 I=1,N=M
 KMBL13MBS(1)=SYMB(1)=YMB(1)
 151 CONTINUE
 IF(LASHM).LE.1000.1 GO TO 66
 UO 89 I=1,N=M
 FM=M-N,M
 F1=I-1
 89 F10SM(1)=F1J1M+(J360.*F1)/FMN
 CALL SPCT
 90 CONTINUE
 DO 51 I=1,N=M
 S1(I)=Z2.*PI*FRSM(1)
 MBS(1)=Z2.*P1.U/(ISU(1))*S1(I)
 MKSM(1)=Z2.*P*/MLSM(1)
 51 FISM(1)=F13SM(1)*PI/AE0.
 IF(LFSM.LE.1000.).OR.(FNSM.GE.2000.).) GO TO 91
 NM=M-N,M
 UO 90 I=1,N=M
 FSM(1)=FRSM(1)&F10SM(1)=F1JSW(1)
 ARSM(1)=ASW(1).DATA(M1)=ASW(1)
 90 CONTINUE
 C PRINT DATA
 WK1(1)=.3021 TITLE
 J02 FORMAT(141+10K,20A4)
 WRITE(*,*)
 91 CONTINUE
 C PRINT DATA
 WK1(1)=.3021 TITLE
 J02 FORMAT(141+10K,20A4)
 WRITE(*,*)
 92 FORMAT(1/104,
 1SSMLISING OF ENVIRONMENTAL AND BUOY CHARACTERISTICS)
 WRITE(*,7)
 7 FORMAT(5A,1b)OCEAN CONDITIONS)
 WRITE(*,8)
 * FORMATT(1X,15HUNFACE WAVE=.5X,3MFRE(CPS).4X,3MHML(FT).2X,
 1MHML(FT).3X,3MHML(FT).2X,3MHML(FT))
 JO 31 I=1,N=M
 31 WRITE(*,9) FRSM(1),ASW(1),F10SM(1),MLSM(1),MKSM(1)
 9 FORMAT(1X,.410.2+10.4)
 WRITE(*,10)
 10 FORMAT(1X,15HCURRENT PROFILE,.5X,3HDEPTM(FT).2X,3HCURR(KT))
 JO 32 I=1,N=M
 32 WRITE(*,11) Y(1),CUK(1)
 11 FORMAT(120X,2F10.2)
 12 FORMAT(1X,15HFLUID DENSITY=.F10.4X,3HSL/NUFT)
 WRITE(*,13)
 13 FORMAT(1/54,15HSURFACE MOTION-.9MFRE(CPS).3X,7HKA(FT).3X,
 17HKA(FT).1X,10MPHDE(0E0))
 JO 33 I=1,N=M
 JS WRITE(*,14) FSH(1),AKSM(1),AYSM(1),F10SM(1)
 14 FORMAT(120X,4F10.4)
 WRITE(*,15)
 15 FORMAT(1/1X,29GENERAL CABLE CHARACTERISTICS)
 WRITE(*,16)
 16 FORMAT(1/2X,3HKA(OFF.2X,3HKA(FAC.1M14
 WRITE(*,17) AMC-AFAC.1M14
 17 FORMAT(12F10.4,F10.2)
 WRITE(*,18)

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PROGRAM CABUDY    *6/74   OPT=0 ROUND=0 / TRACE   FTN 4.6+4.20      04/19/77   11.49.22   PAGE   4
                                                               CABUDY
16 FORMAT(//5X,16HUBLE PROPERTIES,72X,15HBODY PROPERTIES)      1.2
17 WRITE(6,19)                                              CABUDY 1.23
18 FORMAT(1X,3HNUM,3X,7HLEN(FT),2X,3HDIAMINI,2X,3HCON,5X,3HDT,3X,   CABUDY 1.24
17HMLB,/) .1X,7HM(LSL/),1X,9HT,REF(LB),1X,6MC1(LB),1X,   CABUDY 1.25
21X,6HEP G2,2X8MC1M(LB),1X,6HCDAY(F2),1X,3HCDAY(F2),1X,   CABUDY 1.26
3CHMT(LB),1X,7HNM(LSL),1X,7HNM(SL),1X,7HNM(SL),1X,7HNM(SL),   CABUDY 1.27
10 34 21,NCAB                               CABUDY 1.28
11 WRITE(6,20) 1,FLC(I),OC1(I),CON(I),CDT(I),CM1(I),TRFL(I),   CABUDY 1.29
12 J1,I,J2(I),J3,I,COABK(I),COABV(I),WBD(I),XMBV(I),YMBV(I)   CABUDY 1.30
13 CON1UE                                         CABUDY 1.31
14 20 FORMAT(1X,1.3F10.2,4F8.6,F8.6,F18.2,F10.0,F8.6,   CABUDY 1.32
15 1F8.6,24JF4,3,2F8.6)
16 WRITE(6,95) 1BH                                         CABUDY 1.33
17 55 FORMAT(1X,17HX-LOAD ON BOT WT.,F12.6,1X,2HM,0)          CORR 61
18 WRITE(6,59)                                         CORR 62
19 59 FORMAT(1X,2HSURFACE BUOY CHARACTERISTICS)           CORR 63
20 WRITE(6,52)                                         CORR 64
21 52 FORMAT(1X,4HSUBM(FT),2X,11HMEND L0(LB),2X,10HCDAX(FTSO),2X,   CORR 65
11HMIX(TEN(LB))                                     CORR 66
22 WRITE(6,53) SUMM,TWZ,COASX,TBYMX                CORR 67
23 WRITE(6,54) 12,4J                                     CORR 68
24 53 FORMAT(1X,12,4J)                                CORR 69
25 54 SET CONSTANTS AND CONVERT DATA               CABUDY 1.34
26 EPSTA=0.00016EPOTM=0.00016EPR=0.00001
27 RAD=180./PI
28 QELTA=0.00000001
29 10 41 1S1,NSM
30 1S1,NSM
31 HSM(I)=1.0,PI/FSM(I)
32 PI*H(I)*PI*FSM(I)*PI/100.
33 F1=F1*PI*FSM(I)*PI/100.
34 41 CON1UE
35 1L5u1
36 42 1S1,MUR
37 GCF(I)*GCK(I)*1.6470
38 MBOI=MBOINBU
39 FCX=0.
40 1L5u1
41 SYSD=0.
42 IF(LITERLE,0) GO TO 93
43 C PERFORM INITIAL CALCULATIONS FOR ITERATION CASES
44 100 10,1,NCAB
45 101 TOTL=TOTL+1,FLC(I)
46 102 JMH=-1000,VMH=1000,
47 103 DO 96 I=1,NCUR
48 104 96 ACYI=MCYI+FLC(I)*WC1(I)+WBD(I)
49 105 TOTL=9.
50 106 10,1,NCAB
51 107 TOTL=TOTL+1,FLC(I)
52 108 JMH=-1000,VMH=1000,
53 109 DO 94 I=1,NCUR
54 110 94 IF (CCF(I)>-.7,VMX,VMX+CCF(I))
55 111 IF (CCF(I)<-.1,VMH,VMH+CCF(I))
56 112 IF (CY(I).GT.TOTL) GO TO 102
57 101 95 COM1WH,
58 102 IF (VMH.LT.-0.) GO TO 103
59 103 VMX=1.05*VMX
60 104 VMH=1.05*VMH
61 105 VMN=0.95*VMN
62 106 GO TO 93
63 107 VMX=0.95*VMX
64 108 VMH=1.05*VMH
65 109 VMN=0.95*VMN
66 110 CALL ITLWA
67 111 93

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PROGRAM CABUDY T=74 OPT=0 ROUND=0 TRACE FIN 4.6.429 04/19/77 11:49:22 PAGE 5

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238   C CALCULATE STEADY-STATE VARIABLES
      F1KSI=1.
      CALL LURISUBM(CU)
      UCF1=CU
      TDA=0.5*SPRHO*GDA3X*CCF1*ABS(CCF1)
      TIX=TDX*FIN
      IF(I1R1.LT.-3.) TIX=0.5*PRHO*ABS(GDA3X(NBD))*CCC1*ABS(ICCF1)*TBH
      IF(I1R1.LT.-8.) TIX=NE0*(NCA-1)
      TIX=0.5*TIX*TIP1TY
      PHIS=ALAN2(L1A,-11Y)
      TN=M72
      PHISUM=PHIS*AD
      TYS=TYATK3-TIX
      IF(FSMHLE.GE.2000.) GO TO 36
      IF(DIRL1.EQ.0.) GO TO 98
      CALL BUOY
      90 TO 98
      * CONTINUE
      XA=0.5*TA-SUBM*XPAB=0.5*PA=0.
      JO=0, I=1,M,M
      KA=KA+ASHM11*CON(FISM11)
      YA=YA-TYSM11*SIN(FISM11)
      XPA=XPMHSM((I*ASHM11)+DIF(FISM11))
      YPA=YPMHSM((I*ASHM11)+AVSM11)*COS(FISM11)
      * CONTINUE
      99 CONTINUE
      XA*XAS/YA=YASR=0.*ASE=0.
      IF(ITER.EQ.1) GO TO 204
      WRITE(6,116)
      116 FORMAT(1A,30MINITAL VALUES AT TOP OF CABLE)
      WRITE(6,117) XA,YA,XPA,YPA
      117 FORMAT(1X,3HAA,F10.4,5X,3HYA=,F10.4,5X,4HYVA=,
      1F10.4)
      45 WRITE(6,65) MC
      65 FORMAT(1H1,5X,1BHRUN NUMBER,1J)
      WRITE(6,66)
      66 FORMAT(10X,26HSTEADY-STATE CONFIGURATION)
      WRITE(6,67) TIN,TIN,0.1
      68 FORMAT(1A,4H1A,4H00E+2A,9MS REF(FT),
      1H1E6,67)
      67 FORMAT(1A,4H00E+2A,9MS REF(FT),
      1X,9MS STRIFT),3X,SHV(IF1),3X,SHV(IF1),3X,7HTEM(LB),3X,9HPHIS(OEG),
      JS=0
      WRITE(6,69) JS,SR,SE,XC,YC,TEM,PHISD
      69 FORMAT(16.14,6F10.2)
      204 JS=1
      ILST=NOAB
      IF(I1R1.LT.-8.) ILST=NBD
      C COMPUTE STEADY-STATE CONFIGURATION
      00 71 L=IN,ILST
      IC=1
      IF(I1R1.LT.-3.) IC=NCAB-1
      START=0.
      JRG=0.5*DHD*(DC1(IC)/IC)*CON(IC)
      JRAT=0.5*DHD*(DC1(IC)/IC)*COT(IC)
      MPUL=NO(IC)
      THF=THEF(1)
  
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PROGRAM CABUDY	7/4/74	OPT=0 ROUND=0 INACE	FTN 6.6426	04/19/77	11.49.22	PAGE
1C1=C1(C1)				CABUDY	222	
TEX1=1./C21(C1)				CABUDY	223	
JPA=FC1(C1)/2.				CABUDY	224	
00 81 11+1,2				CABUDY	225	
Y(11)=TEM				CABUDY	226	
Y(12)=PHIS				CABUDY	227	
Y(13)=XG				CABUDY	228	
Y(14)=YC				CABUDY	229	
Y(15)=SE				CABUDY	230	
CALL KUMHERIS,SH,YB,EP,TIA,SPA,START,MGX,EPQ,STAT)				CURR	64	
FIND1=1.				CABUDY	232	
TEM(11)=PHIS=Y(12)*XC*Y(13)*YC=Y(14)*SE=Y(15)				CABUDY	233	
PHISD(Y(12)*RAD)				CABUDY	234	
IF (ITER.GE.1) GO TO 205				CABUDY	235	
WRIT1(6+63) IJ+SR,SE+KC+YC+TEM+PMISO				CABUDY	236	
205 IFL(11=1) 81./2.81				CABUDY	237	
72 IC=IC				CABUDY	238	
TEM1=IJI+TEM				CABUDY	239	
PHID1=IJI+PHISU				CABUDY	240	
61 CONTINUE				CABUDY	241	
C COMPUTE TENSION AND ANGLE AT BODY				CABUDY	242	
CALL CUNIVC,CB)				CABUDY	243	
IB=16				CABUDY	244	
IF (UBK1,I-J,0,J) IB=IC-1				CABUDY	245	
IF (UBK1,LE,0) GO TO 144				CABUDY	246	
FBK=(S*RMH+ABS(CDABK1(18)) +C*ABS(CB))				CABUDY	247	
IF (IC-EQ.0) FBK=FX*FBK				CABUDY	248	
IF (IB=0) HNUJ=URDTM-FD1				CABUDY	249	
143 FBY=ABU(18)				CABUDY	250	
FB1=SM(IPH1)*TEM*FBX				CABUDY	251	
FB2=-CO3(IPH1)*TEM*FBY				CABUDY	252	
TEM=SUKT((IB1*BI+TB2*I82))				CABUDY	253	
PHISATAN2(I82,-I82)				CABUDY	254	
PHISUPH1>RAD				CABUDY	255	
IF (IC-EQ.-94) ASL0I=XC				CABUDY	256	
I=IC,ED,NDJ Y1,B1=YC				CORR	89	
IF (IC-EQ.0) Y1,B1=YC				CORR	90	
IF (IC=0,GE,1) GO TO 71				CABUDY	257	
144 WRIT1(6+63) IJ+SR,SE+KC+YC+TEM+PMISO				CABUDY	258	
74 CONTINUE				CABUDY	259	
THNK1=TEM>IN(PHIS)				CABUDY	260	
TYER=+TEM-OS(IPH1)				CABUDY	261	
IF (ITER.GE,1) GO TO 207				CABUDY	262	
WNS1(he73)				CABUDY	263	
75 FORM1(V)				CABUDY	264	
WRITE(6,75) THWRK				CABUDY	265	
76 FORM1(X,2,3MK-COMPONENT OF TENSION=F10.2,1X,2MK)				CABUDY	266	
76 FORM1(X,3MK-COMPONENT OF TENSION=F10.2,1X,2MK)				CABUDY	267	
IF (ITER.LE,0) GO TO 211				CABUDY	268	
IF (ITER.LE,0) GO TO 211				CORR	91	
76 FORM1(X,3MK-COMPONENT OF TENSION=F10.2,1X,2MK)				CABUDY	269	
76 FORM1(X,3MK-COMPONENT OF TENSION=F10.2,1X,2MK)				CABUDY	270	
76 FORM1(X,3MK-COMPONENT OF TENSION=F10.2,1X,2MK)				CABUDY	271	
76 FORM1(X,3MK-COMPONENT OF TENSION=F10.2,1X,2MK)				CABUDY	272	
76 FORM1(X,3MK-COMPONENT OF TENSION=F10.2,1X,2MK)				CABUDY	273	
76 FORM1(X,3MK-COMPONENT OF TENSION=F10.2,1X,2MK)				CORR	92	
76 FORM1(X,3MK-COMPONENT OF TENSION=F10.2,1X,2MK)				CORR	93	

PKG2R3N LABOURY 7/6/76 OPT=0 ROUND=0 / TRACE F1W 4.6+420 04/19/77 21.49.22 PAGE 7
 1.6X+6H2Y+6A+4H1ENB,8X+4H1D+4X+
 14H1B8X+4X+4H1TY+8X+4H1GB,8A+4H1YUB-
 16X+4H1YS
 545 VCA+4H1A+4H1A
 IFLITER.EU(1) XXLNLBLT
 IFLITER.EU(1) YL+YLBT
 MK/7E16.200) ICS.YCRAJXX.RIV.TEN.PHISQ.TMORX.TVERT.XXL.VVL.SV150
 208 FORMATT15.10F12.5)
 ICS.AC5.91
 IF(IKS+GE.150) STOP
 GO TO 93
 211 IF10IA.GE.0.) GO TO 62
 619=DIR
 TIV=INERT
 TIV=THORX
 TEN=SEAR1(YIV+TIV+TIV+TIV)
 THIS=ATAN2(TIV+TIV)
 YMX=IX-0.S RHO*COAS X*CCF1*AB5(CCFS)
 WRITE(6,56) YMX
 56 FORMAT(1A,125)REVISED VALUE OF WIND LO=F12.4+1X.2HLS)
 60 TO 93
 42 13 (ABS(PH10)) .LY.360.) GU TO 45
 UO 83 1+1*40A0
 PH10IJ=PH10JS(1)
 TENSIL1),TENSIL1;
 6.3 CONTINUE
 65 CONTINUE
 FNIA=1.
 DO 63 I=1,10CA
 UC=0.1(I)/2.
 ON11=0.5*PHO*CDN1)*UC*FLC11)
 JK11=0.5*RHO*CD11)*UC*FLC11)
 PH10IJ=PH10JI*P1/100.
 ANK11=ANG-AFC.PI/DC+(DC/4.)*FLC11)/FN11
 GM11=GM11*FLC11/FN11
 STC11=STC11*FLC11/FN11
 66 CONTINUE
 725 TNS=(TENSIL1)*SIN(PH0SS1/RAD)
 C TNS=TENSIL1*(COS(PH0SS1/RAD)
 COMPUTE X AND Y BASED ON ANGLE AND TENSION AT NODEMENT
 360 DO 64 IC=1,NCAB
 641 G11,I,G21,I
 IF(TENSIL1.LE.0.00001) TENSIL1=0.00001
 TMIGATE1(1,IC)=TNSF(IC)/C11(IC)
 STR=AS(1,IC)+IC21-1.,FMTRC
 FLV(F11,IC)(1,*SIR)
 X1IC1=MA-F1*SIM(PH11(IC))
 Y1IC1=Y1*FLC11*PH11(IC))
 XP1(IC)=XP1(IC)*SYP1(IC)*VP1(IC)
 XAK1(IC)
 YAV1(IC)
 XPP1(IC)=XP1(IC)*YP1(IC)=YP1(IC)
 365 CNTINUE
 NUE1=2**NED0
 NODE2=NUE1
 IF(PSM1.GE.2000.) NUE2=NUE1+3
 NDE=2**NUE2
 366

PROGRAM CABUDY 7470 OPT=0 ROUND=0 / TRACE FIN 4.66420 06/19/77 11.49.22 PAGE 6

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400         I1=0.
        WRITE(6,11)
11 FORMAT(1X,0,5X,24MCABLE INITIAL CONDITIONS)
        WRITE(6,12)
22 FORMAT(1X,4HNODE,2X,BMPHI(0E6),3X,7HTEN(L3),4X,5SHX(FT),5X,
15SHY(FT),5X,0MHV(FT/S),2X,0MHV(FT/S),
0D 36 1,0,MLAB
13-1 .
J6 WRITE(6,?1) I,PMOD(I),IEN(I),X((I)),Y((I)),WPI(I),VP(I)
23 FORMAT(1I,1J,0,XF10.4,F10.2,2F10.4)
        WRITE(6,?4)
24 FORMAT(1V/5X,16HTIME INFORMATION)
        WRITE(6,?6) TIM0,DT1
06 FORMAT(1X,22HINITIAL TIME INTERVAL=.F10.4,1X,1HSEC,3X,
11HTIME STEP=.F10.4,1X,3HSEC)
        WRITE(6,?5) DT1,DT2
25 FORMAT(12X,11HTOTAL TIME=.F10.4,1X,3HSEC,3X,10HTIME STEP=.F10.4,
11X,3HSEC)
        WRITE(6,?7)
27 FORMAT(1M1,10X,42HCOMPUTED CABLE SYSTEM MOTIONS AND TENSIONS)
        WRITE(6,?8)
28 FORMAT(1/2X,3HMMH,3X,6H(SEC),2X,7MDT(SEC),3X,5SHX(FT),5X,
15MH(FT),5X,0MAK1FT/S),2X,BMP(FT/S),1X,5MKPP(FSS),1X,
2MHPP(F/S),5X,7MTEW(LB),1X,7MFI(DEC),1X,6HSTRAIN,
11X,6HSTP(1/5))
110P=0
111A=TINVA-JELTA
111B=TINVA-JELTA
111C=OT1
111D=OT2
111E=1.0;START=0.
00 261 4&1,7508
00 1G1 1=1,100
Y0=(1+3*4(1+4*4*(1-2)*XP(I))
Y1=(1-1)*(1)AV0(4*1)=YP(I),
101 CONTINUE
111F=MUE2.EQ.NODE1 GO TO 114
Y0=MUE-2*ZTP1S8(MUE-1)*SY1SY0(MUE-3)=ZET1
111G IF (LT.GT.LN1) OT=OT2
111H LT=1.0;LN1=1.AND.LT.LE. LN2=. START=0.
111I CALL KUTER(INDELT,Y0,EPDM OT,START,MCK,EPK,DTM)
111J COMPUTE SURFACE MOVE MOTIONS
111K A2=0,2YM=0,
      HSMW=0.4YSMP=0,
      ASWP=0,YSWPP=0,
10 161 LN=1,NSM
      MKSF=MSK3((LN1)*XS1-SIG1(LM)*T1*FISM1(LM))
      XS1=COL(MKSFBSSXSF$SIM(MWSF))
      XS2=XSME(SM1(LM)*GMSF
      Y5=Y5-N-SM1(LM)*SMXSF
      X5NP=XSM1(LM)*SIG1(LM)*ASW1(LM)*SMXSF
      Y5P=YSM1(LM)*ASW1(LM)*GMSF
      ASWPP=XSMPP-SIG1(LM)*2*ASM1(LM)*GMSF
      Y5MP=YSPP+SIG1(LM)*2*ASM1(LM)*SMXSF
161 CONTINUE
      WRITE(6,62) TT,MCA,XSM,YSM,XSMP,YSMP,XSWPP
162 FORMAT(1X,6AVEF8.4,F6.6,2F10.2,4F10.0)

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PROGRAM CABUDY 74/76 QPT=0 ROUND= / TRACE FIN 4.6+428 04/19/77 11:40:22 PAGE 9

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 468 IF (NUE2.EQ.NOE1) GO TO 121
      STUV<VA140>./PISSYPP/ASYPPI100./PI
      HR11(6123) TT.MCUXSI.ZETI.XPSI.ZTPX.KPNSI.ZETPP.SYD.SYPPD
      125 FORMAT(IX,4HB0Y,F8.4,F8.6,2F10.2,4F10.4,1D7.2,F8.6,
      11.5MSYPP,F8.2,1K,4D/SS)
      121 (6E X=X(1)-X$DEL(Y=Y(1))-YS
          DPF=MP11,XSPDYP,MP11,MSP
          FLS=SQR(DELX*OELX*OELY*DELY)
          F10S=ATM21-OELX*OELY*180./PI
          EPS=(FLS*FLC(11))/FLC(11)
          EPS=(OELX*DAP+OELY*OY1)/FLS*FLC(11)
          TENS=C(11)*(A85(EPS1)*C(21)-1.)*EPS*REF(11)+CIM(11)*EPSP
          IF (TEN1..LL-TRIN) TENS=TRIN
          F10S=(-OKEP/-C3)+OELX*(OELX*DAP*OELY*DYP)/FLS*3)/
          1C0*(F10S*PI/180.)
          FPN=S(FPS-180./PI
          M1,c16125) (OP,TT,MCK,XSYS,XSP,TENS,F10S,F10S*FPS,EPSP
      125 FORMAT(2E11.1,2E11.1,2E11.1,2E11.1,2E11.1,2E11.1,2E11.1)
      00 187 1.2,RCAN
      OELX=X(1)-K(1-1)*OELY*Y(1)-Y(1-1)
      OAP=M(11)*M(11)*3DYPW*(1-YP(1-1))
      FLS=SQR(DELX*OELX*OELY*DELY)
      FLUS=ATM21-OELX*LELT)*180./PI
      F35=(FLS*FLC(11))/FLC(11)
      EPS=(OELX*OAP+OELY*OY1)/FLS*C(11)
      TENS=C(11)*(A85(EPS1)*C(21)-1.)*EPS*REF(11)+CIM(11)*EPSP
      IF (TENS..LE.1H) TENS=TRIN
      F10S=(-OKEP/FL3)+OELX*(OELX*DAP*OELY*DYP)/FLS*3)/
      1C0*(F10S*PI/180.)
      F10S=F10S*180./PI
      11=1
      WRITE(6,29) 11,TT,MCK,X(11)+Y(11),XP(11),YP(11),AEP(11),VNP(11).
      21 TENS,F10S,F10S*EP,EPSP
      29 FORMAT(2E11.1,2E11.1,2E11.1,2E11.1,2E11.1,2E11.1,2E11.1)
      107 CONTINUE
      IF (TT.GE.10TT) GO TO 1000
      FIRST=100.
      201 READ(MUC)
      1000 COPY I NUE
      SIUP
      ENQ
  
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1

SUBROUTINE STATE	76778 SUBROUTINE STATE	77N 6.0.6.624	8K 19327 21.4.9.22	PAGE
1	SUBROUTINE STARTED IN DERI COMMON/HSP/URACH/DRACT/APUL/TAF+TC1,YE,X1 DIMENSION FIN(1),DER(15) IF (TC11,L,0,0000001) FIN11=0.00000001 CALL CUR11(N11,1,C1) CMORPHOS(FIN11,2,TC1 GTAN=ATAN(FIN11)-TRF1/TC1,1+(TC11-L)*((FIN11)-TRF)/TC1 SIRN=ABS(ATAN(1))-TRF1/TC1,1+(TC11-L)*((TC11)-TRF)/TC1 17 DEK1511=1.0 GNDRAG=(1.414213562373095) PI=DEK1511*GNDRAG*CHORH0AUSCHORNH1 DER11=1-GNDRAG*(FIN12)^2*WPL DER12=1-(1-SIN(FIN12))^2*WPL1/(1-FIN11) DER13=1-SIN(FIN12)^2*WPL1 DER14=1-COS(FIN12)^2*WPL1 NEXT11=1.0 RETURN END		GARDY 416 CABUDY 417 CABUDY 418 CABUDY 419 CABUDY 420 CABUDY 421 CABUDY 422 CABUDY 423 CABUDY 424 CABUDY 425 CABUDY 426 CABUDY 427 CABUDY 428 CABUDY 429 CABUDY 430 CABUDY 431 CABUDY 432	1
5				
13				
19				

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SUBROUTINE DYNH 747% 0F1=9 POUND=*/ TRACT FTU 6.6+420 04/13/77 11:49:22 PAGE 1
1      SUBROUTINE JYHAI(17,FI,DF)
2      DIMENSION JEL106, F1(106)
3      COMMON/H1/A1SH1(120),A1SH1(120),F1SH1(120),MSH1(120),SIG1(120),
4      IF1SH1(120),N3M
5      COMMON/M04/L150, C2(130),I1R7(150),J1R7(150),O1R7(150),A1N(150),
6      COMMON/M05/I1T50,I1T50,I1C(50)
7      COMMON/HB1/R1V,V1GCCF,F5M1,MX,TYS,SUMH,RTH,YCH,
8      18IN,BG,MAS,COAH,CAHS,Y,VPA
9      COMMON/L05/A1D1S1,A1D1S1,X1P1,S1P1,X1L1,Z1P1,S1P1
10     COMMON/H05/X1H1(50),Y1H1(50),Z1H1(50),X1B1(50),Y1B1(50),Z1B1(50)
11     COMMON/H05/X1H1(50),Y1H1(50),Z1H1(50),X1B1(50),Y1B1(50),Z1B1(50)
12     1YP151,2YP151,3YP151,4YP151,5YP151,6YP151,7YP151
13     COMMON/S1P1,S1P1,S1P1,S1P1,S1P1,S1P1,S1P1
14     COMMON/S1P1,S1P1,S1P1,S1P1,S1P1,S1P1,S1P1
15     COMMON/S1P1,S1P1,S1P1,S1P1,S1P1,S1P1,S1P1
16     COMMON/S1P1,S1P1,S1P1,S1P1,S1P1,S1P1,S1P1
17     M0L=2,M0E2,M0=NCAB-1
18     JO 5 151,M0E2
19     S 01 (2*(I-1))=F1(2*(I-1))
20     JO 6 1=M0
21     ((1)*F1((I-1))>XP1(I))>F1((I-2))
22     ((1)*F1((I-1))>XP1(I))>F1((I-2))
23     F1(I)=F1((I-1))+XP1(I)+F1((I-2))
24     CONTINUE
25     IF(F1>MILE-20000.) GO TO 15
26     XSI=F1(MDE-5)>MPSIDE(MDE-5)
27     ZEI=F1(MDE-1)>Z37P>DE(MDE-3)
28     YAN=F1(MDE-1)>S1P1>DE(MDE-1)
29     NSMS1=NS1-N1Y,SY1
30     XSP=XPS-R1Y*S1P1
31     Y30YCGZET1R1X+S1Y+R1Y
32     YSP-Z1P1+R1X*S1P1
33     GO TO 12
34     X1=0.15*SUMMAXSP+0.8YSP+0.
35     JO 11 F1=45M
36     NSTF=-NS1/3.141592653589793
37     AS=XPSMS11+COS(NS1TF)
38     VSYS=NS1MS11*SIN(NS1TF)
39     XSP=XPSMS11+AYSMS11+COS(NS1TF)
40     YSP=YSPMS11+AYSMS11+COS(NS1TF)
41     XAAS1A=VS1AP+XSP*YAP+1SP
42     IC=1
43     FLN=FLC(IC)
44     QELY=Y11-YA
45     QELY=QELY(-DELX,DELY)
46     FIN=ATM2(-QELX,DELY)
47     OX=XP11-XAP+DYP11-MAP
48     CSF=COS(IVISSHF-SIN(IFIV))
49     EPSB=1DELY/CSF-F1M/F1M
50     EPP0=1DELY*D1P0+DELY*D1P1/(SQW1(DELX**2+DELY**2)*FLM)
51     TB=C111*A2*EPG0*(IC211)-1.)*EPS3*IRL1(11)*C1W1(11)*EPPO
52     IF(17.1LE.TMIN) T=t-MIN
53     TEMP=TEMP+IV
54     IF(LABS(ITEMB).GE.-10YMAX) TEMP=10YMAX+TEMP/ABS(ITEMB)
55     DO 41 IC=1,NCAB
56       GM=AMHC11*GM11C
57       CALCULATE EQUATIONS FOR CORNER ELEMENTS

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PAGE 2

SUBROUTINE UNIT	76/76	OPT+0 ROUND0 / TRACE	FTN 4.6+020	96/19/77 11:49:22	PAGE
I	IF (IC>GE, N2AB) GO TO 43 FC=AFCSF63SF+SW*STA-18 FCAB=1.08FC01.0	CABUDY 469 CABUDY 470 CABUDY 471 CABUDY 472 CABUDY 473 CABUDY 474 CABUDY 475 CABUDY 476 CABUDY 477 CABUDY 478 CABUDY 479 CABUDY 480 CABUDY 481 CABUDY 482 CABUDY 483 CABUDY 484 CABUDY 485 CABUDY 486 CABUDY 487 CABUDY 488 CABUDY 489 CABUDY 490 CABUDY 491 CABUDY 492 CABUDY 493 CABUDY 494 CABUDY 495 CABUDY 496 CABUDY 497 CABUDY 498 CABUDY 499 CABUDY 500 CABUDY 501 CABUDY 502 CABUDY 503 CABUDY 504 CABUDY 505 CABUDY 506 CABUDY 507 CABUDY 508 CABUDY 509 CABUDY 510 CABUDY 511 CABUDY 512 CABUDY 513 CABUDY 514 CABUDY 515 CABUDY 516 CABUDY 517 CABUDY 518 CABUDY 519 CABUDY 520 CABUDY 521 CABUDY 522 CABUDY 523 CABUDY 524 CABUDY 525 CABUDY 526 CABUDY 527 CABUDY 528 CABUDY 529 CABUDY 530 CABUDY 531 CABUDY 532 CABUDY 533 CABUDY 534 CABUDY 535 CABUDY 536 CABUDY 537 CABUDY 538 CABUDY 539 CABUDY 540 CABUDY 541 CABUDY 542 CABUDY 543 CABUDY 544			
60	I	IF (IC>EQ, -1) FC=2.0 IF (IC>EQ, -INCAB-1) FCB=2.0 IC1=IC+1 DELx=IC1)-X-(IC1*DELy=Y(1:IC1))-Y(IC) UXP=X(1:IC1)-XP(1:IC1)/SWY(1:IC1)-YP(1:IC) GSP=COS(IFI)*SSWF*SH(IFIV) FLM=FC(GIC1) EPSB=(DELx*UXP+UYL)*YP/(ISQR((DELx*2+OELy*2)*FLM)) EPSB=(DELx*UXP+UYL)*YP/(ISQR((IC1-1)*EPSB*TREF(IF1))+ 1C1*WIF1)+EPSB IF (TBLLE, TBLN) TB=THIN TB=TB*SWFACT*SWF TV=A*CSF+T0*CSF AMP=0.01*WMP*0. DO 37 IS=1,MSW UKV=MSW(IIS)*Y(IC1) IF /ABS(UKVK)-GE, 5, 1 GO TO 38 UKH=SFMSKSM(IIS)*WIC1*SIG(IIS)*T*SFISW(IIS) 37 YMP=YMP*ASH(IIS)*SIG(IIS)*EXP(-UVT)*SFISUUKXSF) 38 CALL CURTIC1(JU) YR(X)=PCU-XP(1:IC1) VR(Y)=IP-TC1 FR(SQRT(VR*VR*VR*VR)) VRMA=VR*CSFA+VR*SFHFA VRMB=VR*CSFO*VR*SMF VRTA=VRK*SWA*VR*CSFA VRTB=VRK*SMF*VR*CSF OMA=9.708N1C1*VR*B*ABS(VRMA) OMB=9.708N1C1*VR*B*ABS(VRMB) OTA=9.708N1C1*VRTA*ABS(VRTA) UTB=9.708N1C1*VRTB*ABS(VRTB) DMX=CA*DNA*CSF*AVPCB*OND*CSF OMY*CSFA*DNA*SMFA*CB*OND*SMF OTX=CA*DA*SMFA*CB*OND*SMF UTY*CA*DA*CSFA*PCB*OND*CSF SF((GDBSK1C1),J1,-1) OR ((DABY((IC1),LT,0.)) GO TO 81 CO=ADSK1C1)/(ICABY((IC1)+0.00000001)) IP((GDAE,LE,0,-52,OR,(GDAE,GE,2,)) GO TO 71 UBA=9.708N1C1*VR*VR RVA=9.708N1C1*VR*VR RVE=9.708N1C1*VR*VR	469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540 541 542 543 544		
65	FIV=ALAN2-*JEI,X*OLLY, UXP=X(1:IC1)-XP(1:IC1)/SWY(1:IC1)-YP(1:IC) GSP=COS(IFI)*SSWF*SH(IFIV) FLM=FC(GIC1) EPSB=(DELx*UXP+UYL)*YP/(ISQR((DELx*2+OELy*2)*FLM)) EPSB=(DELx*UXP+UYL)*YP/(ISQR((IC1-1)*EPSB*TREF(IF1))+ 1C1*WIF1)+EPSB IF (TBLLE, TBLN) TB=THIN TB=TB*SWFACT*SWF TV=A*CSF+T0*CSF AMP=0.01*WMP*0. DO 37 IS=1,MSW UKV=MSW(IIS)*Y(IC1) IF /ABS(UKVK)-GE, 5, 1 GO TO 38 UKH=SFMSKSM(IIS)*WIC1*SIG(IIS)*T*SFISW(IIS) 37 YMP=YMP*ASH(IIS)*SIG(IIS)*EXP(-UVT)*SFISUUKXSF) 38 CALL CURTIC1(JU) YR(X)=PCU-XP(1:IC1) VR(Y)=IP-TC1 FR(SQRT(VR*VR*VR*VR)) VRMA=VR*CSFA+VR*SFHFA VRMB=VR*CSFO*VR*SMF VRTA=VRK*SWA*VR*CSFA VRTB=VRK*SMF*VR*CSF OMA=9.708N1C1*VR*B*ABS(VRMA) OMB=9.708N1C1*VR*B*ABS(VRMB) OTA=9.708N1C1*VRTA*ABS(VRTA) UTB=9.708N1C1*VRTB*ABS(VRTB) DMX=CA*DNA*CSF*AVPCB*OND*CSF OMY*CSFA*DNA*SMFA*CB*OND*SMF OTX=CA*DA*SMFA*CB*OND*SMF UTY*CA*DA*CSFA*PCB*OND*CSF SF((GDBSK1C1),J1,-1) OR ((DABY((IC1),LT,0.)) GO TO 81 CO=ADSK1C1)/(ICABY((IC1)+0.00000001)) IP((GDAE,LE,0,-52,OR,(GDAE,GE,2,)) GO TO 71 UBA=9.708N1C1*VR*VR RVA=9.708N1C1*VR*VR RVE=9.708N1C1*VR*VR	469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540 541 542 543 544			
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105	C COMPUTE DRAG AND ADDED MASS COEFFICIENTS FOR CIRCULAR DISK ACWDRD, TO REPORT MAOC-AE-7-20 1. IF (ICABY((IC1),LT,0.)) GO TO 85 JIAH=SF((IC1),J1,-1) IF (ABS(XPPIC1)) .LE. 0.0000001 XPP1C1=0.0000001 Y2AD=SF((IC1),J1,-1)/(XPPIC1)/(XPPIC1) SV2=SF((IC1),J1,-1)*2.5Y2R*(0.01AHC) AMV1C1=AMV1C1*2.5Y2R*(0.01AHC) COAHC1=COAHC1*2.5Y2R*(0.01AHC)	469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540 541 542 543 544			

SUBROUTINE DINA 747% OPT=8 ROUND= / TRACE PTN 4-6429 06/19/77 11:49:22 PAGE 3
 115 GO TO 71
 85 DINA=SIGN(-b*GDA8(Y(IC)/(1.12*PI)),
 IF IABS(YP(IC))>LE,0.000001) YP(IC)=0.000001
 Y2D=ABS(YP(IC)*Y(P(IC)))/(YP(IC)*Y(IAM))
 SY2D=RT(Y2D)
 IF I>Y2-6,JKUL SY2=DKUL\$IF(SV2.LE.OKLL) SV2=0.0LL
 GDA8(YC)=(PI*DA((I+2*I-4)*2,2)/SY2
 YHRAIC=TRMBCIGA*2*SY2*RHODIAPO*3
 DBX8=5*HO*GDA8(IC)*RK*ASIN(VR)
 DBY8=5*HO*GDA8(IC)*VRY*ABS(VRY)
 71 FRAK(DNA,JTK+08:
 IF IL=EQ(NBU) FX=-1+TBH
 FYTY*DN*DTY*DTY*0.0*SFCA*WNC(IC)*0.5*FCB*WTC(IC)*WBD(IC)
 FIAB*5*(IC-AHNC)*FCG6(IC)+FCA*AHNC)*CSA*CSFA*
 IFCB*AHNC*(CSF)*XMBV(IC)
 51*Y8=5*FCFA*6MIC)+FCG6(MIC)+FCA*AHNC)*CSA*CSFA*
 IFCB*AHNC)*SM*SFV1(IC)+XMBV(IC)
 FX8=0.5*IFCA*AHNC*SM*SFVA*FCA*AHNC)*SFV*CSF
 XPP(IC)=FIY*FXFK*FYI/(IFIY*FYI-FK*FFK)
 YP(IC)=FIY*FX-FK*FYI/(IFIY*FYI-FK*FFK)
 DE8*IC-2)=XPP(IC)*BDE8*IC1*YPP(IC)
 135 41 CONTINUE
 43 CONTINUE
 IF(FSM1.GE.20000) GO TO 51
 RETURN
 C CALCULATE COMMON TERMS FOR SURFACE BUOV
 51 XMP=0.AVP=0.
 DO 52 IS1=1,NSW
 UAY=MHSIS)*SUBM
 IF IABS(UAY)<6.E-5.) GO TO 52
 UKS=NMHSIS*XS1-S(GIS),(+FISH(WIS)
 AMPA=XMP*SM(WIS)*SIGIS)*EXP(-UKY)*SIN(WUXSF)
 TNPAYMPASW(WIS)*SIGC(WIS)*EXP(-UKY)*COS(WUXSF)
 52 CONTINUE
 VS=XMP*CF1-(XPSL-BG*SYPL)
 VS=YMP-ZIP1
 THA-TENB*SIN(FVB)*THV*TENB*GOS(FVB)
 UBS=0.SPHO*COASK*VS#*ABS(VSK)
 FXUOKATHA*IMM
 DA=0.SPHO*COASY*VS#*ABS(VST)
 F=0.BY*TMY-TYS*AGS#*ETI
 FSY*UBH*-BL)+(ATY*STI*RTK)*TMY-(RTX*STI*RTV)
 1*THK-RM*THA
 1*FSM1*W-E-30000) GO TO 61
 C CALCULATE DIFFERENTIAL EQUATIONS FOR SPAR BUUV
 160 PSIC=P51
 FKRX=0.BFKRKY=0.BFRKRS=G*PSI*SYI
 DO 95 I=1,NSW
 SGI=MKH(I)) XSI-S(GI)) TO FISM(I)
 FMKR=FMKR-2*SIG((I+2*SM((I+0.005*(SGF))+QSO(I))
 FRKS=FRKS+Z*SIG((I+2*SM((I+0.005*(SGF))+QSI(I))
 55 FRKRY=FRKRY-5M11)*SM(SGT1)*COS(SGT1)*QS1(I)+RGSO(I)
 JEMUE=-1*(0.1*NP52)*(FX*FRKRY)+SIG*((SIG(I)+2*WVN*USA(I))+RGSO(I))
 1*BVKH*(BNH*PS)-PSIC*21
 DE INDE 2*(BVKH*(FSY*FRKS)*PSIC*(FX*FRKR))+(BVKH*(BIN*PS2))-
 1*PSIC*21
 JE INDE -2)*(F+F*FRKR)/BVH2
 CABUDY 546
 CABUDY 547
 CABUDY 548
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 CABUDY 571
 CABUDY 572
 CABUDY 573
 CABUDY 574
 CORR 1.21
 CABUDY 575
 CABUDY 576
 CABUDY 577
 CORR 1.22
 CORR 1.23
 CABUDY 581
 CABUDY 582
 CABUDY 583
 CABUDY 584
 CABUDY 585
 CORR 1.26
 CORR 1.27
 CABUDY 587
 CORR 1.28
 CORR 1.29
 CABUDY 590
 CABUDY 591
 CABUDY 592
 CABUDY 593
 CORR 1.30
 CORR 1.31
 CORR 1.32
 CORR 1.33
 CORR 1.34
 CABUDY 597

SUBROUTINE DYN4
 7077, OPT=NRD:IND=0 / TRACE
 FTM 4.66421
 06/19/77 11:49:22
 PAGE 4

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  GO TO 64
  61 GOM/IND CALCULATE DIFFERENTIAL EQUATIONS FOR SPHEROIDAL BUOY
  62          SPC=SPP
  175          FRKRX=0,AFRKS=0,VGRSY=0
  176          DYS=0.
  UC 65 I=1+IND
  SSFLAM=SM((1-XS1-SI4(1))+(1-SM(1))
  *COSISGF1)
  YAKK=MM*MM*SI4((1)*2*ASMI1)*COSISGF1)
  FAKRS=FRKS*(SPC+PS1)+SIG11*2*ASMI1
  *COSISGF1)
  FAKRS=FRKS*(SPC+PS1)+SIG11*2*ASMI1
  *COSISGF1)
  180          FRKR=FRKR-5b((1) SIN(SG1)*(-IG(1)*2*WHz*AG50)
  65 FRKAY=FRKAY-5b((1) SIN(SG1)*(-IG(1)*2*WHz*FRKS-UTS))
  DE INUT=0) n(BNPF*(FAFRKR)+SPC*(FSV+FRKS-UTS))/_
  185          LIBNM*BNP-SPC=0
  186          ISYFRKR1/BWNZ
  JE INDE=0) =ISYFRKR1/BSY
  DE INDE=0) LIBNM*(FSY+FRKS-OTS)*SPC*(FX+FRKRX)/(BNM*BNP-SPC**2)
  188          XPPSI=OC(MJF-0.182E1PP*DE1INDE-2)*SYPP*DE1INDE)
  189          RETURN
  EN3

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SUBROUTINE CUR
R/R_N OPT=8 ROUNDS=8 TRACE
FTN 4.6+420

STATEMENT NUMBER	STATEMENT	PACR
1	COMMON/NCU/V(1),CCF(10),MCUR,FIRST,VCX	
2	IF (FIRST,L1,0,0) I=1	CABUY 617
3	IF (V-L1,0,0) GO TO 09	CABUY 618
4	IF (V,LT,V(MCUR)) GO TO 38	CABUY 619
5	IF (V,GT,V(L1)) .AND. (V,LE,V(V(I+1))) GO TO 30	CABUY 620
6	IF ((V,GE,V(L1)),.AND.,(V,LE,V(V(I)))) GO TO 49	CABUY 621
7	IF ((V,GE,V(V(I+2))),.AND.,(V,LE,V(V(I+2)))) GO TO 58	CABUY 622
8	I=1	CABUY 623
9	IF (V,LE,V(V(I+1))) GO TO 38	CABUY 624
10	I=1	CABUY 625
11	GO TU 70	CABUY 626
12	3W CU=CCF(I)+((CCF(I+1)-CCF(I))/(VV(I+1)-VV(I)))*(V-VV(I))	CABUY 627
13	L=VCK	CABUY 628
14	RETURN	CABUY 629
15	60 I=I+1	CABUY 630
16	GO TU 38	CABUY 631
17	50 I=I+1	CABUY 632
18	GO TU 38	CABUY 633
19	60 CUCCF(I)=V-X	CABUY 634
20	RETURN	CABUY 635
21	99 CU=CCF(MCUR)-VCX	CABUY 636
22	RETURN	CABUY 637
23	END	CABUY 638
		CABUY 639
		CABUY 640

SUBROUTINE SPEC1	1474	OPT=0 ROUND=0 / TRACE	FTN 4.6.6.4.20	04/19/77	11.49.22	PAGE
1		SUBROUTINE SPEC1	CABUDY	641		
		COMMON /MOP/ASHM(20),FRSM(20),MSM,MTRC	CORR	642		
		G=32.28P+3.145926	CABUDY	643		
		FLLFRSM(1)	CABUDY	644		
		FULLFRSM(12)	CABUDY	645		
5		FNSM=MSM	CABUDY	646		
		DELTAN=2.*PI*(FUL-FLL)/FNSM	CABUDY	647		
		MP2=PI*FLL	CABUDY	648		
		IF(ASM(11,66,2000).LT.60) GO TO 20	CABUDY	649		
		C EIGENVALUES FOR FREQUENCY SPECTRUM	CABUDY	650		
		10 MSG=ASM(11)-1888.	CABUDY	651		
		ARE=0.001*(G38-33.56/(MSG*HSC))	CABUDY	652		
		JO 11 1=1,NSM	CABUDY	653		
		F1=1	CABUDY	654		
		MH=MPP*DELTAN	CABUDY	655		
		WH=0.5*(IPD+AM)	CABUDY	656		
		MPAIN	CABUDY	657		
		FRSM(11)=MH/(2.*PI)	CABUDY	658		
		SSH=A*EXP(-d/(MH**2))/MH*S	CABUDY	659		
		ASMI1=2*RT((SSM*DELTAN))	CABUDY	660		
		11 CONTINUE	CABUDY	661		
		12 RETURN	CABUDY	662		
		C COMPUTE MORE OTHER FREQUENCY SPECTRA	CABUDY	663		
		20 CONTINUE	CABUDY	664		
		END	CABUDY	665		

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SUBROUTINE BUOY      T=70   OPT=0 MOUND=0 / TRACE          FTN 4.6+420      04/19/77  11.49.22    PAGE  1

1      SUBROUTINE BUOY
COMMON/RD0/NMD,P1,G,CCF1,FSH1,TMK,TXS,TVS,SUBH,RTY,RTX,RMY,YCG,
1d10,dc,MAS,CUAS,COAS,RA,TA,XPA,YPA
COMMON/RD1N/XPPSI,ZETPI,SYPP,XSI,ZETI,SYI,XPSI,ZTP1,SYPI
COMMON/SPS/PS0,PS1,PS2,PS3(20),PS4(20),PS5(20),PS6(20),PS7(20),
COMMON/RD1/AXSM(20),ATSM(20),FTSM(20),MSM(20),SIG(20),WSM(20) .
1FLSM(20),NIM
COMMON/RSR/ASM20,FSRM20,NISW,NIFAC
COMMON/AMER/BUP,SPF,NGYBS,AVIP,DTM,WMXK,WVHZ,AB
COMMON/UNI/UNI201,UNI1201
UNISSION 2M9(1201) 0M11201
COMMON/MIT/PHASL,XRL,YRL,TIL,VHL,VHM,VWH,3C1,IIR,ICS,NDOT,ORCAH,TIV
1.SYSLU
DIMENSION MH(20),FS(20),FM(20),FP(20),FPS(20)
1FL10,GE,2) GO TO 31
HEAD(5,1) CUABY,MAS,RMX,RTX,RTY,YCG,01M
HEAD(5,1) ASJ,ZET1,STUD,XPS1,ZTP1,SYPO1
HEAD(5,1) ASJ,ZET1,STUD,XPS1,ZTP1,SYPO1
IFINRC,LE,0) GO TO 211
2 L  CONVENT METRIC INPUT TO ENGLISH UNIT,
CUASY=3.281*3.281, CUAS=CUASH=4.523, RMY=3.281*MMY
RTX=3.281*3.281, RTY=3.281*RTYUGA,3.281*YCG8,BIN=0.737*BIN
XSI=3.281*3.281,ZET1=3.281*ZET1,XPSI=3.281*ZTP1
211 CONTINUE
1 FORMAT(0F10.4)
SYSLSYN=PL/100,
SYPL=SYPO1,PI/100.
25  NRIT(6,3)
3 FORMAT(/5K,23HSURFACE BUOY PARAMETERS)
NRIT(6,3)
4 FORMATT(X,LUMCDO(F11.0),2X,6HMF(10),6X,7HMNY(10),3X,7HRTX(10)).
JU  1.3X,7HRTY(10),2X,7HMDY(10),2X,10HMSLFT30),2X,6HMF(10),2X,
2.9H21AL(FT),2X,6HSY1(DEC),1X,6HXY1(F/S),
12X,9HCVY1(F/S),
MKT(6,5) COAST,MAS,RMY,RTX,RTY,YCG,RIM,XSI,ZET1,SVO1,XPSI,
1ZTP1,SYPU1
5 FORMATT(11F10.4)
J5  dMS=MAS/3
J1  TV=1,TVSMA,
1  IF(15M1.GE.3000) GO TO 101
L  COMPUTE COFFICIENTS FOR SPAT BUOY
BMY=2*dM>+TVSG
BVR=8MS*1.4*QRHO/J,J=((8MS*TVS/LJ)/(IRHG*PI*AYSH1(NSM)))**1.5
UN011)=0.
11 JN1=10,0.
45  PH0=0.,AP1=0.,SPH2=0.
MS2=M2M2
JU 2  LS1,M3M2,2
H=AM(M1S1)-AS(M1S)
ALFA=(AYSM(1,2)-ATSM(1,1))/H
FAU(H,ALFA+0,1)(G,ALFA)
ALP2=2.*ALFA-1.*ALP1*(ALFA+1.)*2*ALP=ALFA*(2.-ALFA)
32*ALASHM(1,2)/AS2*ASMSH(1,2)*AS1*ASMH(1,2)
Y2=Y2-Y2*Y1G=Y1-YGGAGG=Y-VCG
PNUPNU+FAJ=ALP2>2*ALP1->ALP1+S
PN1=PN1+A-(ALP2*Y2G*S2+A*PI*V1G*2*ALP*YGG*S)
PN2*PN2*FAJ=(ALP2*Y2G*S2*2*ALP*Y1G*2*3*ALP*Y1G*2*5)

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SUBROUTINE BUOY    74/74   OPT=0 ROUND=0 / TRACE          F7IN 4.6+628      04/19/77 11:49:22      PAGE   2

DO 14 IM=1,NW
  NK=1+IM*(NW-1)
  YK1=Y-TNK*SM1(NK)
  YK2=Y-TNK*SM1(NK)
  EXP1=EXP(YK1)*EXP(YK1)*EXP(YK2)
  EXP2=EXP(YK2)*EXP(YK2)*EXP(YK2)
  QN1=(IM1*IM1*IM1)*FAC*(ALP2*EXY2*EX2*ALP1*EXY1*SI*ALP*EXY*SI)
  QN2=(IM1*IM1*IM1)*FAC*(ALP2*EXY2*EX2*ALP1*EXY1*SI*ALP*EXY*SI)
  JALP1*GCEKF*SD
14  CONTINUE
2  CONTINUE
  FAC=FACC*PHD*RHOB*PSI*FACC*PHD*RHOB*PSI*FACC*PHD*RHOB
  DO 6 I=1,MN
    Q5111=AC1*QH0111*RH0
    Q5111=FACC*QH1111*RH0
    H658=RH06*AKSM111
    YC8=PS1*PS0
    8607LB
    NM2511=(EXYK2*QH5)/PS0
    IF(I11E.GE.1).AND.(ICS.GE.2)  GO TO 401
21  WRITE(6,21)
22  FORMAT(7X,2PMC0MANTS FOR SPAR BUOY)
  MRAKE(6,261) FACC
26  FORMAT(1A,20NMINITIAL RATIO T /PG=,F10.5)
  WRITE(6,23) PS0*PS1*PS2
23  FORMAT(1X,3MPa,F10.5*X,4MP1=,F10.5*X,3MP2=,F10.5)
  MRAKE(6,24) (050(1),1=1,MSK)
  MRAKE(6,25) (053(1),1=1,MSK)
24  FORMAT(1A,3HQ=,12F0.5/X,12F10.5)
25  FORMAT(1A,3HQ=,12F0.5/X,12F10.5)
  MRAKE(6,29) MN2*RG50
29  FORMAT(1X,5H0M2=,F10.5*S1,5HRCG50=,F10.5)
  GO TO 401
401 CONTINUE
C  COMPUTE COEFFICIENTS FOR PROLATE OR OBLATE SPHEROID
C  OF TAYLORIAN BOD-1AKSM11
  A=BSHADFT/A
  IF (MA.GT.0.1) GO TO 141
  FM5=0.075*MA/0.1
  FMH=1.2*0.9/MA
  FMPS=1.2*0.9/MA
  FMPS=0.3
100  GO TO 121
141  IF (MA.GT.10.) GO TO 161
  MH(1)=0.15MH(2)=0.25MH(3)=0.35MH(4)=0.45
  MH(5)=0.75MH(6)=0.95MH(7)=1.05MH(8)=1.15
  MH(9)=2.0MH(10)=3.0MH(11)=5.0MH(12)=7.0
  MH(13)=8.0
  FS(1)=0.07648FS(2)=0.163FS(3)=0.2068FS(4)=0.31
  FS(5)=0.1978FS(6)=0.449FS(7)=0.568FS(8)=0.622
  FS(9)=0.784FS(10)=0.864FS(11)=0.898FS(12)=0.933
145  FS(13)=0.96
  FH(1)=1.06*SFH(2)=5.04*SFH(3)=1.226SF(4)=2.005
  FH(5)=1.325*SFH(6)=0.968*SFH(7)=0.936*SFH(8)=0.464
  FH(9)=0.323*SFH(10)=0.104*SFH(11)=0.0928*SFH(12)=0.049
  FH(13)=0.028
  FP(1)=1.276FP(2)=0.158FP(3)=0.3128FP(4)=0.117

```

SUBROUTINE BUOY T4/T6 OPT=0 ROUND=0 / TRACCE FTN 4.6+28 04/19/77 11.49.22 PAGE 3

```

115      FP(5)=0.0358FP(6)=0.0038FP(7)=0.00FP(8)=0.0731
         FP(9)=0.2723FP(10)=0.9938FP(11)=3.6588FP(12)=0.-20
         FP(13)=28.0
         FP(5)=0.3FP(12)=0.2646FP(13)=0.2378FP(14)=0.0177
         FP(5)=0.115FP(15)=0.00398FP(16)=0.005FP(17)=0.191
         FP(19)=0.115FP(18)=0.7918FP(16)=-0.7848FP(11)=-1.5658FP(12)=-2.38
         FP(5)=13)=-3.75
116      I=1
117      IF (MA.LE.MM(I+1)) GO TO 143
         I=1
118      GO TO 147
119      CONTINUE
         FK5=FS(1)*(IF(S(I)+1)-FS(I))/((MM(I+1)-MM(I))*(MA-MM(I)))
         FK4=FS(1)*(IF(I+1)-FM(I))/((MM(I+1)-MM(I))*(MA-MM(I)))
         FK3=FS(1)*(IF(I+1)-FP(I))/((MM(I+1)-MM(I))*(MA-MM(I)))
         FK2=FS(1)*(IF(S(I+1)-PS(I+1)/(MM(I+1)-MM(I))))*(MA-MM(I))
120      GO TO 121
C USE SPAR BUOY STRIP THEORY FOR W/A.GT.10
121      CONTINUE
         FK3=1.-0.4*EXP(-0.09944*(MA-10.))
         FK4=0.028
         FK5=0.028
         FKPS=-J.*MA/5.
122      GO TO 121
123      VOL=2.*PI*UR*T*B*B/3.
         VM1=PI*RR*RR*(DFT*DFT/4.-7.*VCC*DFT/3.)
         FACC=Y1*1./RHO*VOL
         VOL=FACC*VOL*VM1*FACC*VM1
         BG=VM1/VOL
         DOPL=FKP*5*A*YCG*FKS
         API=FKP*A*Z.*VCG*FPS*AYCG*2*FKS
         BMX=BR3*FK3*MM0*VOL
         BMZ=BR3*FK3*MM0*VOL
         BVIP=BIN*API*MM0*VOL
         MMK=11.*FK3*MM0*VOL
         MMZ=11.*FK3*MM0*VOL
         SPF=COPL.*RHO*VOL
         HVBG=RHO*2*VOL*BC
         RGSB=RHO*6*PI*BB/88
         PS1=RHO*VM1
124      IF (LITER.GE.11).AND.(IGS.GE.2)) GO TO 401
         WRITE(6,131)
125      FORMAT(5X,2HCOM,NTS FOR SPHEROIDAL BUOY)
         M17C(4,2,6) = ACC
         WRITE(6,132)
126      1MWOL(CFTT,1X,10MWOL*Y(F1),4X,3MFKS,7X,3MFKM,7X,3MFKP,0,X,
         1MFKP,S,0,4M16S0)
         WRITE(6,133) DFT,0B,MA*VOL,VM1,FKS,FKM,FKP,FKPS,RGSB
127      FORMAT(10F18.4)
128      CONTINUE
129      YSI=YUGAZELI
130      CALCULATE STATIC VALUE OF SY
         ORH=(-TAK-TKS)*(-86)
         SYM=-DMH-Q1X2TS*Q1Y2TS*SYM*TMX
         SY10=6*PS1-R1Y1TS-Q1X1TS
         SY15=15TM/SY10
  
```

SUBROUTINE SUBOV
 TA/T₀ OPT=0 ROUND=0 / TRACE
 FIN 4.6+428
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```

SYISD=SYIS*100./PI
IF(LITER.LT.11) AND(.NOT.GE.2)) GO TO 406
WHITE(0,28) BU
20 FOR AT15X8MFCB=YCGC=F10.4,1K,2MF1)
MRI E(0,27) SYSD
FO'INAT15X8MFCB=YCGC=F10.4,1K,3M0C()
27 FO'INSTEADY SYI=F18.4,1K,3M0C()
404 /* (YU116.E.368.) SYI=SYIS
C COMPUTE YA,XA,XA,TPA
KA=XA,MH=X-NV*SYI
TA=VS*WV*X*SYI
XA=XPsi-RTY*SYI
YPA=ZTPi-RTX*SYI
RETURN
END
  
```

175 180 185

SUBROUTINE ITERA 76/76 OPT=0 ROUND=0 / TRACE FIN 4.3+420 86/19/77 11:49:22 PAGE 1

```

1      SUBROUTINE ITERA
2      COMMON/RCUR/V(18),CCF(140),MCUR,FIRST,VCK
3      COMMON/MIN/PHSL,XRL,YRL,TIL,VNK,VNM,BCV,IYER,ICS,W007,DRCSTH,TIV,CORR
4      IYVSD
5      IF(IITER.LE.0) RETURN
6      GO TO 1181,201,380) ITER
7      C   PERFORM ITERATIONS FOR FREE FLOATING CASE
8      AND IF(IICL.GE.2) GO TO 111
9      10.  VNM=0.5*VNMX*VNW
10.  TIV=BCL
11.  TIV=MY-BCTY
12.  TITM=0.
13.  UNFACT=7
14.  TMAX=.05*TMW=.1.8
15.  PREFERENCE.
16.  RETURN
111  ERX=-TIV*SIN(P1*ISL)
17.  ERY=STL*(COS(P1*ISL))
18.  ERASORT(ITER=ITERA*P1*ERY*ERY)
19.  RATIO=ANSTAY/W007)
20.  W011=.05*(W007)
21.  IF(LAB1:DRCSTH).LE.W011) DRCSTH=W011
22.  IF(LAB1:DRCSTH).GE.ABS(W007) DRCSTH=ABS(W007)
23.  RATIO=ANSTAY/DRCSTH)
24.  VTPVYC
25.  VTTEMP=IV
26.  IF((RATIO.LT.-0.01).AND.(RATIO.LT.-0.01)) GO TO 501
27.  IF((RAT103.LT.0.01)) GO TO 121
28.  IF((EXX-LT.0.?) GO TO 115
29.  112  VCA=0.5*(VCK*VNW)
30.  VNKA*TEMP
31.  RETURN
32.  115  VCA=0.5*(VCK*VNW)
33.  VNKA*TEMP
34.  RETURN
35.  121  IF((IY-EY/PAFRV).GT.-0.5) DMFACT=0.5*DMFACT
36.  IF(IYR/PRKX).GT.-0.7) DMFACT=1.5*DMFACT
37.  PREKERY
38.  TIVTEMP=DMFACT*ERY
39.  IF(ER1.LT.0.) GO TO 125
40.  122  IF(IY.GE.-11YMX) TIV=0.5*(TIVTEMP.TIVMX)
41.  TIV=MY/TIVTEMP
42.  TMAX=TIVTEMP
43.  TIVTEMP=TEMP
44.  GO TO 127
45.  125  IF((IY.LE.-11YMN) TIV=-0.5*(TIVTEMP+TIVMN))
46.  TMAX=TIV/TIVTEMP
47.  TIV=TIVTEMP
48.  TIVTEMP=TEMP
49.  127  VNKA=(TMAX+0.05)*VCK
50.  VNKA=(VNKA-0.05)*VCK
51.  WRITE(6,191)
52.  191  FORMATT/
53.  RETURN
54.  201  CONTINUE
55.  C   PERFORM ITERATIONS FOR MOORED CASE
56.  IF(L115.GE.2) GO TO 211

```

SUBROUTINE LITERA 74/74 OPT=6 ROUND=0 / TRACE FTM 4.0 60428 04/19/77 11:49:22 PAGE 2

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    TIY=-2.*BCYI
    TIYNN=100.*BCYI
    TIYMK=0.
    RETURN
  60   ABSERVABSI(ERY)
    OBSERVABSI(ERY)
    IF(LABSERV.LT.0.01) GO TO 501
  65   TTEMP=TIV
    IF(ERY) 212,212,210,
  212  TIY=0.5*(TTEMP+TIYNN)
    TIYMK=TTEMP
    RETURN
  214  TIY=0.5*(TTEMP+TIYNN)
    TIYNN=TTEMP
    RETURN
  70   CONTINUE
  301  RETURN
  501  ITER=0
  75   RETURN
    END
  
```

CORR 249
 CORR 250
 CORR 251
 CORR 252
 CORR 253
 CORR 254
 CORR 255
 CORR 256
 CORR 257
 CORR 258
 CORR 259
 CORR 260
 CORR 261
 CORR 262
 CORR 263
 CABUDY 663
 CABUDY 661
 CABUDY 662
 CABUDY 663

```

SUBROUTINE KUTHER    74/74,  OPT=8 ROUND=0 / TRACE          FTN 4.6.6428      04/19/77  11.49.22 - PAGE 1
1           C
1          SUBROUTINE KUTHER(M,I,Y,EPS,H,FIRST,HCYA,DAUY)
1          KUTHER ROTINE REVISED FOR IVODE JAN 30.1964
1          VERSION Y0((106),Y1((106)),Y2((106)),F0((106)),F1((106))
1          IF((FAST))20.10.20
10         HC=M
1          IPLDC=1
1          FIRST=1.
20        LOC=8
1          HC=MC
30        CALL DAUX (T,Y0,F0)
39        DO 48 I=1,M
48        Y1(I)=Y8(I)+(MC/3.)*F0(I)
49        CALL DAUX (T+MC/3.,Y1,F1)
50        Y1(I)=Y8(I)+(MC/6.)*F0(I)+(MC/6.)*F1(I)
51        CALL DAUX (T+MC/3.,Y1,F1)
52        DO 58 I=1,M
58        Y1(I)=Y8(I)+MC/8.*F0(I)+375*MC*F1(I)
53        CALL DAUX (T+MC/2.,Y1,F2)
54        DO 79 I=1,M
79        Y1(I)=Y8(I)+MC/2.*F0(I)-1.5*MC*F1(I)+2.*MC*F2(I)
80        CALL DAUX (T+MC,Y1,F1)
81        DO 86 I=1,M
86        Y2(I)=Y8(I)+MC/6.*F0(I)+66666667*MC*F2(I)+(MC/6.)*F1(I)
87        MC=0
88        DO 110 I=1,N
89        ZZZ=ABS (Y1(I))-A
100       IF(ZZZ) 65,67,67
65       ERROR = ABS(-2*(Y1(I))-Y2(I))
66       IF(ERROR>A)108,108,98
67       ERROR=ABS (1.2--2*Y2(I))/Y1(I)
68       IF(ERROR>EPS)108,108,98
90       CONTINUE
91       KISCIE=24
92       C2A=R2.*KISCIE
93       XN= C2A*REABS (MC)-ABSI (H)
94       IF ((XN) 92) 1,ERROR,I
92       FORMAT(2I1M RELATIVE ERROR AT X= 1P1E12,3,3W 1SF10.6/
93       FIRST = 2.
94       RETURN
95       MC=MC/2.
96       IPLDC=2 + IPLC
97       LOC=2 * LOC
98       MCX=MC
99       GO TO 38
100      IF(ERROR<6.0-EPS)110,110,101
101      INC1
110      CONTINUE
111      IF((MC))
112      Y0(I)=Y2(I)
113      LOC=LOC+1
114      IF((LOC>PLLOC))120,210,210
120      IF((MC)>2.0,130,210
125      IF((LOC>1L0J/2.+2*210,140,210
130      IF((PLLOC-1)>210,210,200
135

```

SUBROUTINE	KUTNER	74/7.	OPT=0	20 . . . / TRACE	FTN 4.6+4.20	04/19/77	11.49.22	PAGE	2
60					CABUDY	921			
					CABUDY	922			
					CABUDY	923			
					CABUDY	924			
					CABUDY	925			
					CABUDY	926			

```

51 MC=2.*MC
1 CC=LOC/2
IPLOC=IPLOC/2
210 IF(IPLOC-LOC)30,220,30
220 RETURN
END

```

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